Free boundary equilibrium code: CEDRES++

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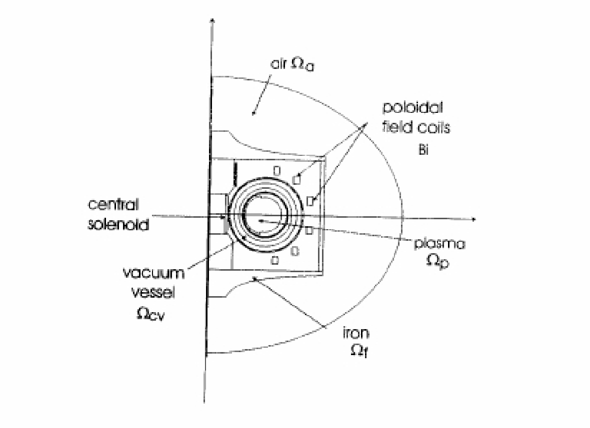
Introduction

- Solve plasma equilibrium problem for an axisymmetric tokamak.
- Free boundary code
- Direct problem : static and evolutive version
- Inverse mode
- V. Grandgirard : "Modélisation de l'équilibre d'un plasma de Tokamak (1999)"

1. Introduction
2. Direct problem
   - Static version
   - Evolutive version
3. Inverse problem

Direct problem

Poloidal cross-section Ω



Static version

Equation on ψ

$$L_\mu \psi = j_\phi$$

where

$$L_\mu \psi = -\frac{\partial}{\partial r}(\frac{1}{\mu r}\frac{\partial \psi}{\partial r}) - \frac{\partial}{\partial z}(\frac{1}{\mu r}\frac{\partial \psi}{\partial z})$$

- L_μ : elliptic operator
  - non-linear in the iron (the permittivity μ depends on $\mathbf{B_p}^2$)
  - linear everywhere else ($\mu = \mu_0$)
- j_ϕ : toroidal component of the current density
  - $j_\phi = 0$ everywhere except in the PF coils B_i and the plasma Ω_p

Boundary conditions

- $\psi = 0$ on (Oz)
- $\psi = 0$ at the infinity

In the iron

$$L_\mu \psi = 0 \text{ in } \Omega_f$$

In PF coils

$$L_{\mu_0} \psi = \frac{I_k}{S_k} \text{ in } B_k,\ k = 1, ..., N_c$$

I_k : total current flowing in the coil B_k.
S_k : cross-section area of B_k.

In the plasma

MHD equilibrium equation $\mathbf{J} \times \mathbf{B} = \nabla p \rightarrow$ Grad Shafranov

$$L_{\mu_0}\psi = j_\phi$$

with

$$j_\phi(r,\psi) = rp'(\psi) + \frac{1}{\mu_0 r} ff'(\psi)$$

$$\Gamma_p = \{M \in \Omega_{cv} / \psi(M) = \psi_b\}$$

$$\Omega_p = \{M \in \Omega_{cv} / \psi(M) \geq \psi_b\}$$

where $\psi_b = max_D\psi$ in limiter configuration or $\psi_b = \psi(X)$ in X-point configuration.

Input

- Currents I_k in the coils B_k, and total plasma current Ip
- Magnetic permeability function $\mu(\mathbf{B}_p^2)$
- Plasma current density : **given analytically** $j_\phi(r,\overline{\psi}) = \lambda j_T(r,\overline{\psi})$

where $j_T(r,\overline{\psi}) = (\frac{r\beta}{R_0} + \frac{R_0(1-\beta)}{r})(1-\overline{\psi}^\alpha)^\gamma$

and $\overline{\psi} = \frac{\psi - \psi_a}{\psi_b - \psi_a}$.

$$I_p = \lambda \int_{\Omega_p} j_T(r,\overline{\psi}) d\Omega$$

or given by point $(p'(\overline{\psi})$ and $ff'(\overline{\psi}))$.

Output

Free boundary plasma equilibrium (ψ and Ω_p).

Numerical method

- P_1 **finite elements**
- Condition $\psi = 0$ at infinity solved using a **boundary integral method**
- Boundary integrals appearing in the finite element method are given analytically (for circular boundary)
- **Size of the computational domain and computation time reduced**
- Non linearities : Picard and/or Newton methods

CEDRES++ code

- Written in C++
- No machine dependent : mesh generated automatically giving the machine description
- ITM integration : can use CPOs as input and output
- Kepler actor available (soon with numparam)

Mesh example

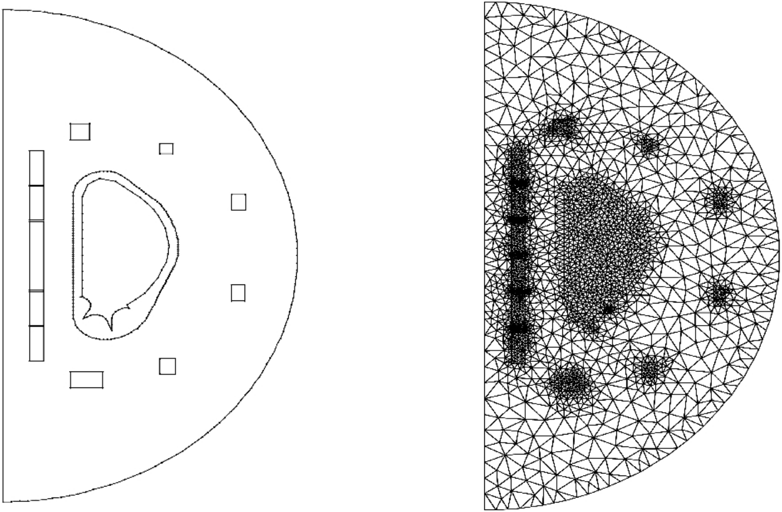
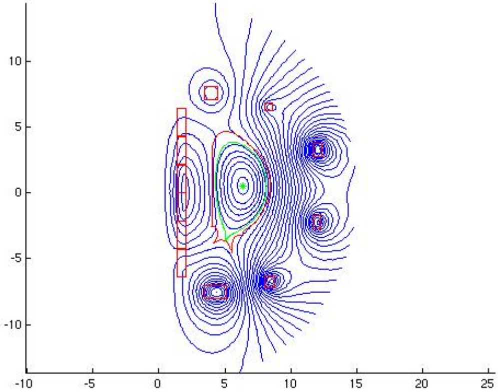


FIG.: Machine description (input) and mesh generated with Triangle

Numerical results : ITER



Equilibrium with iron : Tore Supra

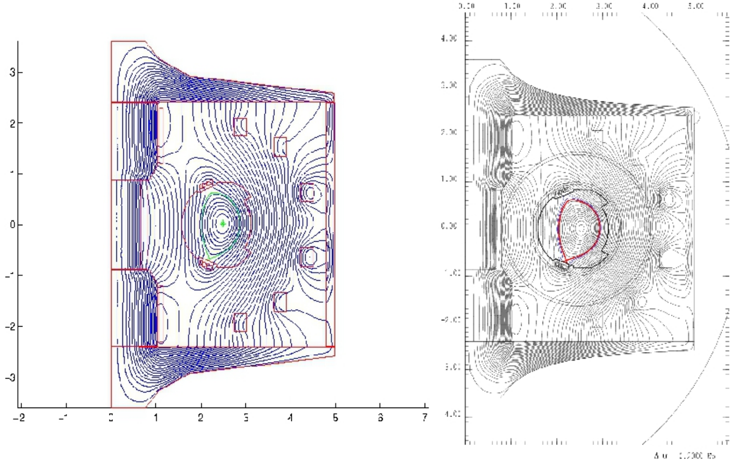


FIG.: Comparaison CEDRES++ (left) - Proteus (right)

Evolutive version

In PF coils

$$V_i(t) = R_i S_i I_i(t) + \frac{n_i}{S_i} \int_{B_i} \frac{\partial \psi}{\partial t}.$$

In each PF coils B_i

$$L_{\mu_0} \psi = \frac{V_i}{R_i S_i} - \frac{n_i}{S_i^2} \int_{B_i} \frac{\partial \psi}{\partial t} dS.$$

In vacuum vessel and passive structures

$$L_{\mu_0} \psi = -\frac{\sigma_v}{r} \frac{\partial \psi}{\partial t}$$

where σ_v is the conductivity.

- Time treated implicitly
- New input : at each time step $V_i(t)$
- Validation in progress
- Example on ITER

Inverse problem

Objectives

Looks for an optimal distribution of current in the PF coils in order to best match a desired plasma shape.

Input

- Caracteristics of plasma boundary (shape, X-point)
- I_k in B_k, $k = m+1, ..., N_c$ fixed

Output

I_k in B_k, $k = 1, ..., m$

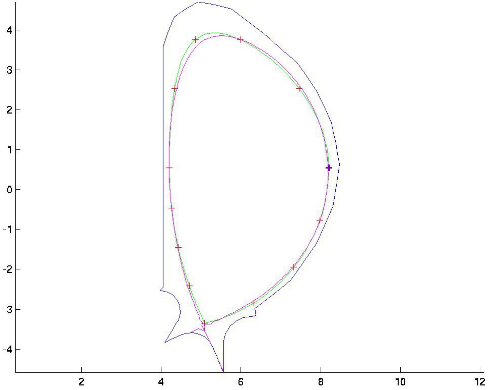
Inverse problem

Minimize the cost function

$$J(I,\psi) = \frac{1}{2}\oint_{\Gamma_d} \alpha(M)[\psi(M) - \psi(M_0)]^2 + \frac{1}{2}\alpha_x \int_{v_x} \frac{1}{r}|\nabla\psi|^2 ds + \frac{1}{2}\sum_{k=1}^{m} k_k I_k^2$$

- Γ_d : desired plasma boundary
- M_0 : a particular point of Γ_d
- I_k : intensity in the k^{th} coil and $I = (I_1, I_2, .., I_m)$
- v_x neighborhood of X-point

Inverse problem : numerical example



Comparison between the desired plasma boundary (dots and green curve) and the plasma boundary obtained by CEDRES++ (pink curve)

Perspectives

- Validation of the evolutive version
- Coupling CEDRES++ with a transport solver (CRONOS)
- Optimization of the applied voltages to achieve a given scenario