Zonal Flows and their stability: The role they play in L-H Transitions and density limits

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OUTLINE OF THE TALK

- GENERAL INTRODUCTION
- SIMULATION RESULTS
- THEORETICAL MODEL
- COMPARISON WITH EXPERIMENTS
- CONCLUSIONS

WHAT SHOULD THEORY/SIMULATIONS EXPLAIN ?

Physics issues

- What physics is responsible for L-H Transitions ?

- Which mode/modes is/are responsible for transport in the edge ?

- How does zonal flow get generated and what is its role in transition?

More pragmatic issues

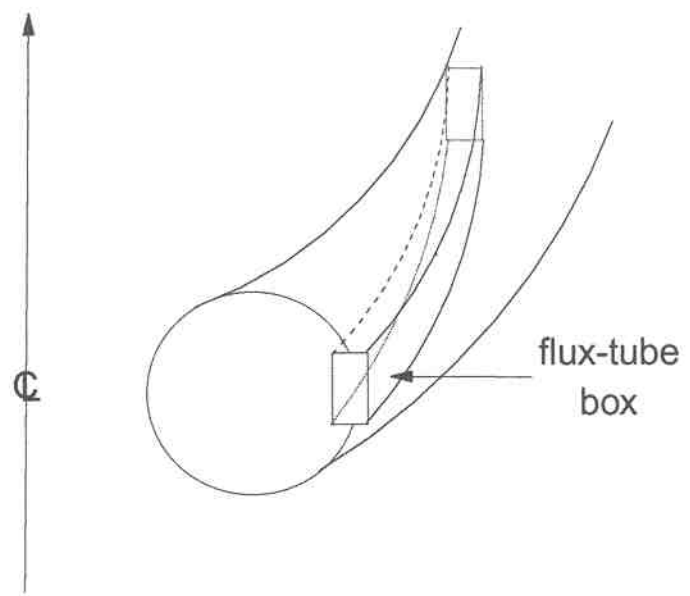
- Why is the power threshold (in most machines) when ∇B drift is away from the X-point 2 -4 larger compared to the case when ∇B drift is towards the X-point ?

- Why does the pellet injection reduce the threshold power for the transition ?

- Can the same physics be playing a role in triggering an H-mode in tokamaks with an imposed electric field ?

3D SIMULATION GEOMETRY

3D NONLINEAR SIMULATIONS OF RBM



BASIC EQUATIONS

A. Zeiler, J, F. Drake and B. Rogers, Phys. Plasmas **4**, 2134, 1997

$$\frac{dn}{dt} + \frac{c_s^2}{\Omega_i L_n}\frac{\partial\phi}{\partial y} + \frac{2c_s^2}{\Omega_i}\vec{b}\times\vec{\kappa}\cdot\nabla(\phi - n) - \nabla_{\parallel}J + c_s\nabla_{\parallel}v_{\parallel} = 0$$

$$\frac{c_s^2}{\Omega_i^2}\nabla_{\perp}\cdot\frac{d}{dt}\nabla_{\perp}\phi - \frac{2c_s^2}{\Omega_i}\vec{b}\times\vec{\kappa}\cdot\nabla p_e - \nabla_{\parallel}J = 0$$

$$\frac{1}{v_A}\frac{\partial\psi}{\partial t} + \frac{c_s^2}{v_A\Omega_i L_p}\frac{\partial\psi}{\partial y} - \nabla_{\parallel}(\phi - n) = \frac{J}{\sigma}$$

$$\frac{dv_{\parallel}}{dt} = -c_s\left[\nabla_{\parallel}p_e + \frac{c_s^2}{v_A\Omega_i L_p}\frac{\partial\psi}{\partial y}\right]$$

$$J = v_A\frac{c_s^2}{\Omega_i^2}\nabla_{\perp}^2\psi$$

$$\nabla_{\parallel} = \nabla_{\parallel 0} + \frac{c_s^2}{\Omega_i^2}\nabla\varsigma\times\nabla\psi\cdot\nabla$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{c_s^2 R_0}{\Omega_i^2}\nabla\varsigma\times\nabla\phi\cdot\nabla$$

$$n = \frac{\tilde{n}}{n_0}, \quad \phi = \frac{e\tilde{\phi}}{T_e}, \quad \sigma = \frac{T_e}{\eta_{\parallel}n_0 e^2}$$

$$v_{\parallel} = \frac{\tilde{v}_{\parallel}}{c_s}, \quad \psi = \frac{\Omega_i v_A}{c_s^2 B_0}\tilde{\psi}$$

BASIC EQUATIONS-CONTINUED

$$\frac{c_s^2}{\Omega_i^2}\nabla_\perp\cdot\frac{d}{dt}\nabla_\perp\phi-\frac{2c_s^2}{\Omega_i}b\times\kappa\cdot\nabla p_e-\nabla_\parallel J=0$$

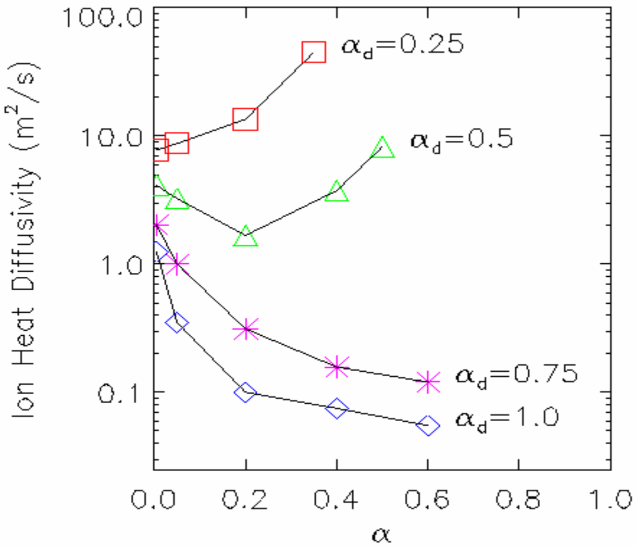
$$x_\perp\to x_\perp/L_0,\ x_\parallel\to x_\parallel/L_\parallel,\ t\to t/t_0 \qquad L_\parallel=2\pi qR,\ n/t_0\sim c_s^2\phi/L_0L_n\Omega_i,\ J\sim\sigma\nabla_\parallel\phi$$

$$1 \quad : \quad \frac{RL_n}{2c_s^2}t_0^2 \quad : \quad \frac{4\pi v_A^2\eta_\parallel t_0L_0^2}{c^2L_\parallel^2}$$

$$\Rightarrow t_0=c_s\left(\frac{2}{RL_n}\right)^{1/2} \qquad L_0=\frac{L_\parallel c}{v_A}\left(\frac{\eta_\parallel}{4\pi t_0}\right)^{1/2}$$

Two dimensionless parameters: (1) $\alpha_D=\omega_* t_0$ (2) $\alpha_{MHD}=\left(\frac{v_A}{qR}t_0\right)^2$

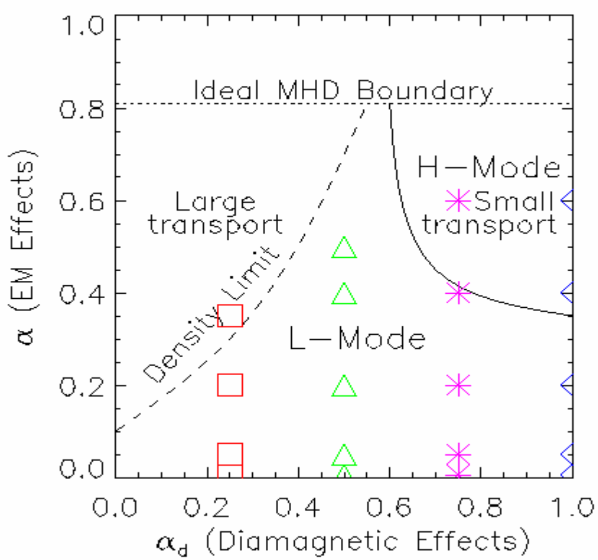
Rogers, Drake and Zeiler, PRL, 81, 4396, 1998



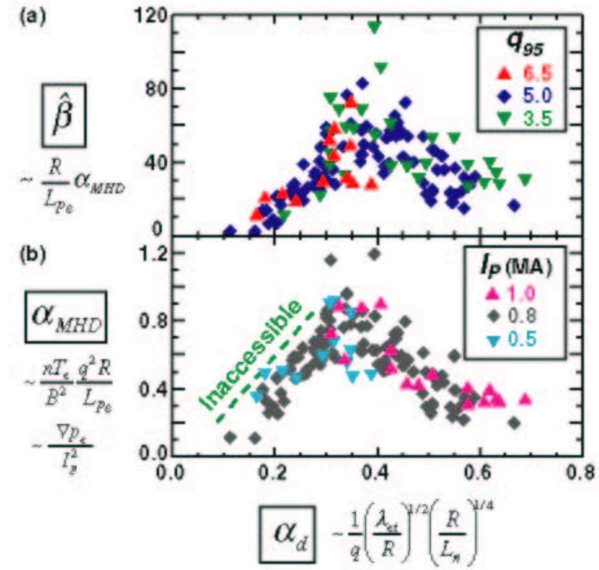
$$\alpha_D = \frac{\rho_s c_s t_0}{L_0 L_n (1+\tau)},$$

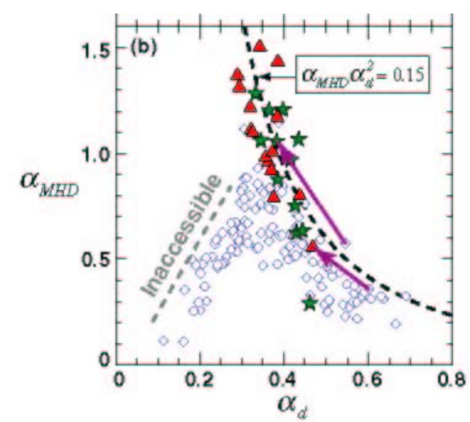
$$t_0 = \frac{(RL_n/2)^{1/2}}{c_s}$$

$$\alpha = \frac{q^2 R\beta}{L_p},$$



LaBombard et al NF 45,1568,(2005)





Rogers, Drake and Zeiler, PRL, 81, 4396, 1998
Simulations with evolving density profile

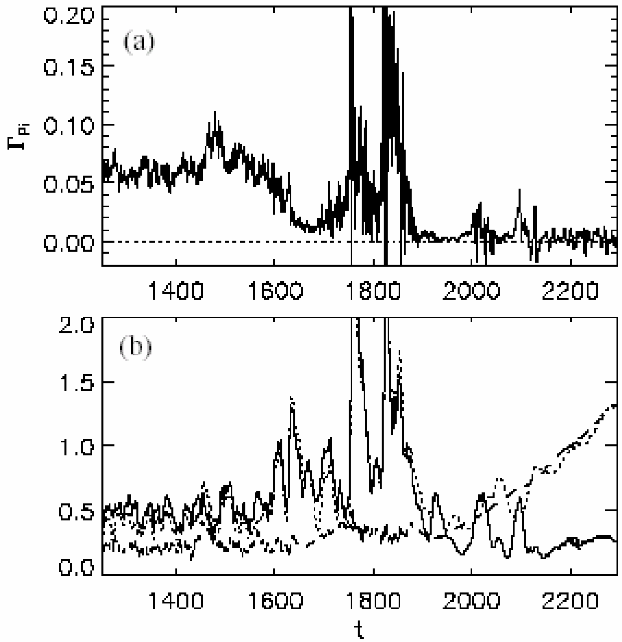


FIG. 4. (a) Γ_{p_i} vs t; (b) $\bar{v}_{iy}$ (solid line); $\bar{v}_{diy}$ (dashed line); $\bar{v}_{Ey}$ (dotted line).

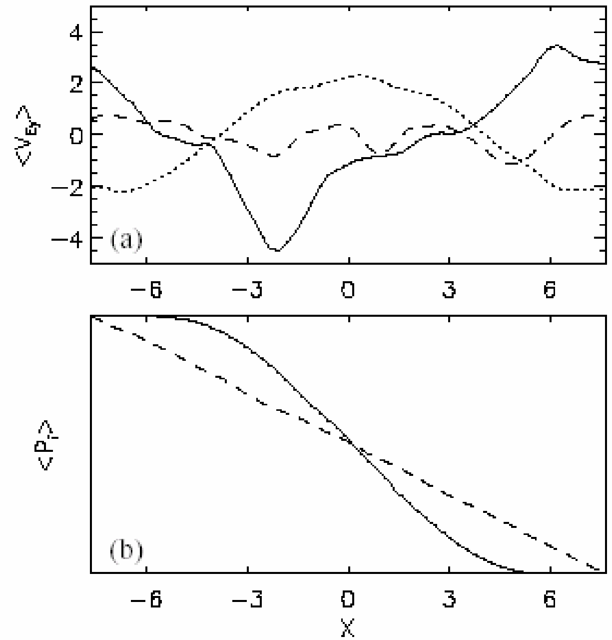


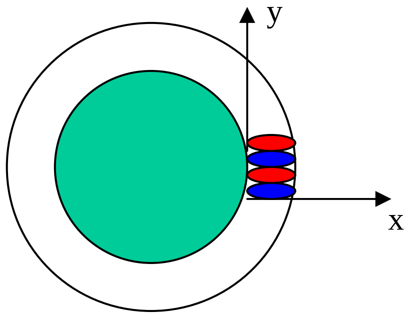
FIG. 5. (a) $\mathbf{E} \times \mathbf{B}$ flows before (dashed line), during (solid line), after (dotted line) transition; (b) early (dashed line), late (solid line) p_i profiles.

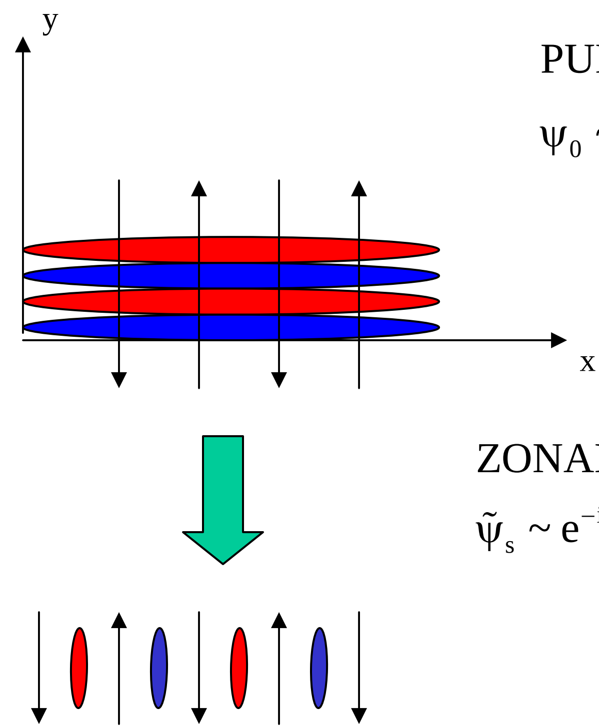
“HINTS” FROM 3D SIMULATIONS

- “Transition” to improve confinement occurs for $\alpha_D \sim 1$,
- L-mode turbulence in the drift wave-like regime (not the resistive ballooning regime)
- Increase in plasma β caused larger anomalous transport even for $\alpha_{MHD} < 1.0$
  Rogers and Drake, Phys. Rev. Lett. **79,** 229 (1997)
- Multiple zonal “layers” which evolve into single layer after transition
- zonal/shear flow present in the L-mode phase
  zonal/shear flow necessary but not sufficient for L-H transitions

IF ZONAL FLOW IS RESPONSIBLE FOR TRANSITION IT IS NECESSARY TO STUDY THE GENERATION MECHANISM IN FINITE BETA PLASMAS

“MODULATION” INSTABILITY





PUMP WAVE

$\psi_0 \sim e^{-i\omega_0 t + ik_y y + ik_\parallel z}$

ZONAL FLOW/FIELD

$\tilde{\psi}_s \sim e^{-i\omega t + ik_x x}$

LINEAR INSTABILITY ANALYSIS

Guzdar et al., PRL, **86**, 15001, 2001, Guzdar et al., PoP

$$\begin{Bmatrix} \phi \\ \psi \end{Bmatrix} = \underbrace{e^{-i\omega t} \begin{Bmatrix} \phi_s \\ \psi_s \end{Bmatrix} e^{ik_x x}}_{\text{"shear" flow}} + \underbrace{e^{-i\omega_0 t} \begin{Bmatrix} \phi_0 \\ \psi_0 \\ n_0 \end{Bmatrix} e^{ik_y y + ik_\parallel z}}_{\text{pump wave}}$$

$$\underbrace{+ e^{-i\omega_+ t} \begin{Bmatrix} \phi_+ \\ \psi_+ \\ n_+ \end{Bmatrix} e^{i(k_x x + k_y y + k_\parallel z)} + e^{-i\omega_- t} \begin{Bmatrix} \phi_- \\ \psi_- \\ n_- \end{Bmatrix} e^{i(k_x x - k_y y - k_\parallel z)}}_{\text{two side bands}}$$

with $\omega_\pm = \omega \pm \omega_0$

“LOCAL” INSTABILITY ANALYSIS (1)

$$\nabla_{\|} \rightarrow ik_{\|} = i(m - nq)/Rq$$

$$(\omega + \Delta)\phi_{+} = i\Gamma_{z}\left[\phi_{0}\phi_{s}M_{A}(k_{x}, k_{y}) + \frac{\omega_{0}}{k_{\|}v_{A}}\phi_{0}\psi_{s}M_{B}(k_{x}, k_{y})\right]$$

$$(\omega - \Delta)\phi_{-} = -i\Gamma_{z}\left[\phi_{0}^{*}\phi_{s}M_{A}(k_{x}, k_{y}) + \frac{\omega_{0}}{k_{\|}v_{A}}\phi_{0}^{*}\psi_{s}M_{B}(k_{x}, k_{y})\right]$$

$$(\omega + i\nu_{\phi})\phi_{s} = i\Gamma_{z}\left[1 - \left(\frac{\omega_{0}}{k_{\|}v_{A}}\right)^{2}\right]\left[\phi_{0}^{*}\phi_{+} - \phi_{0}\phi_{-}\right]$$

$$(\omega + i\nu_{\psi})\psi_{s} = i\Gamma_{z}(k_{x}^{2}\rho_{s}^{2})\frac{\omega_{0}}{k_{\|}v_{A}}\left[\phi_{0}^{*}\phi_{+} - \phi_{0}\phi_{-}\right]$$

“ LOCAL” INSTABILITY ANALYSIS (2)

$$M_A = \frac{(1+\tau)(1+k_y^2\rho_s^2\tau)}{\left(1+\frac{\omega_{*e}}{\omega_0}\tau\right)A} - \frac{\omega_0^2}{k_{\|}^2 v_A^2 A}\left[\frac{1+k_\perp^2\rho_s^2\tau}{1+k_y^2\rho_s^2\tau}\left(1+\frac{\omega_{*e}}{\omega_0}\tau\right) + (1+\tau)\left(1-\frac{\omega_{*e}}{\omega_0}\right)\right] +$$

$$\left[\frac{\omega_0^2}{k_{\|}^2 v_A^2}\left(1-\frac{\omega_{*e}}{\omega_0}\right) - k_\perp^2\rho_s^2\right]\left[\frac{k_x^2-k_y^2}{k_\perp^2} - \tau\frac{k_y^2}{k_\perp^2}\frac{\left(\frac{\omega_{*e}}{\omega_0} - k_y^2\rho_s^2\right)}{1+k_y^2\rho_s^2\tau}\right] / A$$

$$M_B = \left[-\frac{2k_y^2}{k_\perp^2}\frac{1+k_\perp^2\rho_s^2\tau}{1+k_y^2\rho_s^2\tau}\left(1-\frac{\omega_{*e}}{\omega_0} + k_y^2\rho_s^2\right)\right] / A$$

$$\Gamma_z = \frac{k_x k_y c_s^2}{\Omega_i} \qquad \Delta = k_x^2\rho_s^2\omega_0\,(1+\tau)/A$$

“ LOCAL” INSTABILITY ANALYSIS (3)

$$A = 1 + k_\perp^2 \rho_s^2 (1+\tau) - 3 \frac{\left\{ \omega_0 \left[\omega_0 + \frac{2}{3} \omega_{*e} \left(\tau - 1 \right) \right] - \frac{1}{3} \omega_{*e}^2 \tau \right\}}{k_\parallel^2 v_A^2}$$

ω_0 is given by the dispersion relation

$$1 + k_y^2 \rho_s^2 (1+\tau) - \frac{\omega_*}{\omega_0} - \frac{(\omega_0 - \omega_*)(\omega_0 + \omega_* \tau)}{k_\parallel^2 v_A^2} = 0$$

This dispersion relation has three roots, two Alfven waves with effects of diamagnetic drifts and one drift wave with finite beta effects

" LOCAL" INSTABILITY ANALYSIS (4)

Dispersion Relation solved numerically

1. Solve for the "pump" mode eigenfrequency for finite beta drift wave and drift-Alfven wave

$$\Omega_0[1+k_y^2\rho_s^2(1+\tau)]-1-\Omega_0 k_y^2\rho_s^2\hat{\beta}(\Omega_0-1)(\Omega_0+\tau)=0$$

$$\text{where } \Omega_0=\frac{\omega_0}{\omega_*} \quad \text{and} \quad \hat{\beta}=\beta\frac{q^2R^2}{2L_n^2}$$

2. Use the eigenfrequency in dispersion relation for the shear flow/field to calculate growth rate

$$\hat{\gamma}_z=\left[\hat{M}_A(1-k_y^2\rho_s^2\Omega_0^2\hat{\beta})+\hat{M}_B k_x^2\rho_s^2 k_y^2\rho_s^2\Omega_0^2\hat{\beta}-\hat{\Delta}^2\right]^{1/2} \qquad \hat{\Delta}^2=\frac{1}{2}\left(\frac{\rho_s^2}{L_n^2}\right)\frac{(k_x^2\rho_s^2)}{|\phi_0|^2}$$

Five dimensionless parameters:

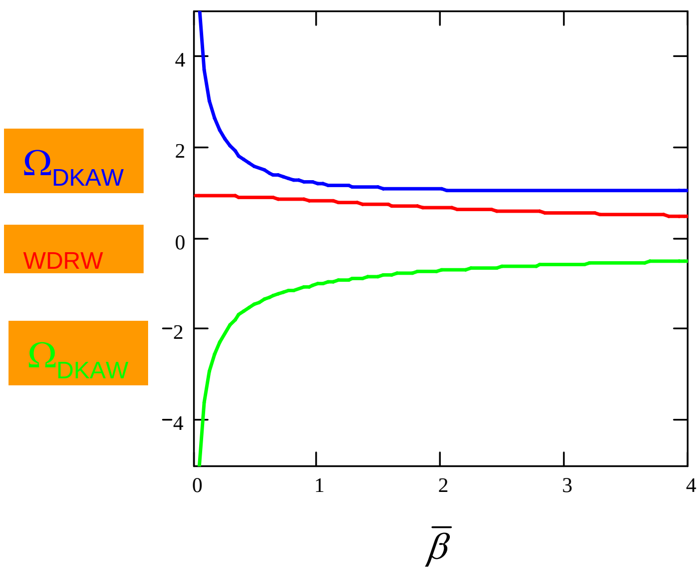
$$(1)\ k_x\rho_s \quad (2)\ k_y\rho_s \quad (3)\ \hat{\beta} \quad (4)\ |\phi_0|L_n/\rho_s \quad (5)\ \tau=\frac{T_i}{T_e}$$

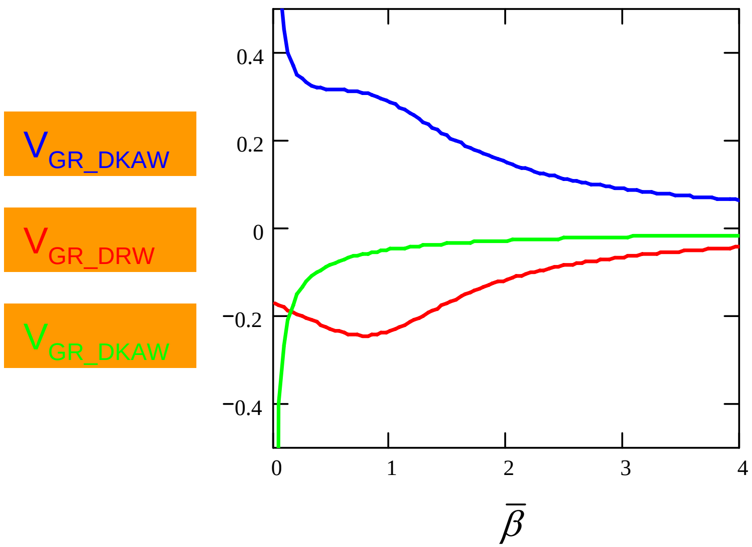
GROWTH RATE FOR ZONAL FLOW/FIELD
$k_y\rho_s$=0.25, $e\phi/T_e = \rho_s/L_n$

Dimensionless Parameters

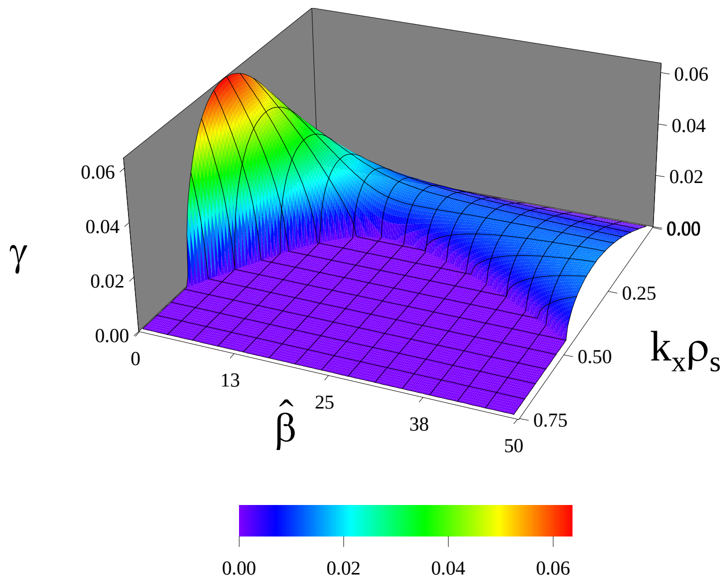
(1) $\mathrm{k_y} = k_y\rho_s$ (2) $\mathrm{k_x} = k_x\rho_s$ (3) $\tau = \mathrm{T_i}/\mathrm{T_e}$ (4) $\Phi = \dfrac{e\phi}{T_e}\dfrac{L_n}{\rho_s}$ (5) $\bar{\beta} = (k_y\rho_s)^2\hat{\beta}$

$\hat{\beta} = \dfrac{\beta}{2}\dfrac{q^2R^2}{L_n^2}$ Parameters: $k_y\rho_s$=0.25, τ=0, $e\phi_0/T_e = \rho_s/L_n$

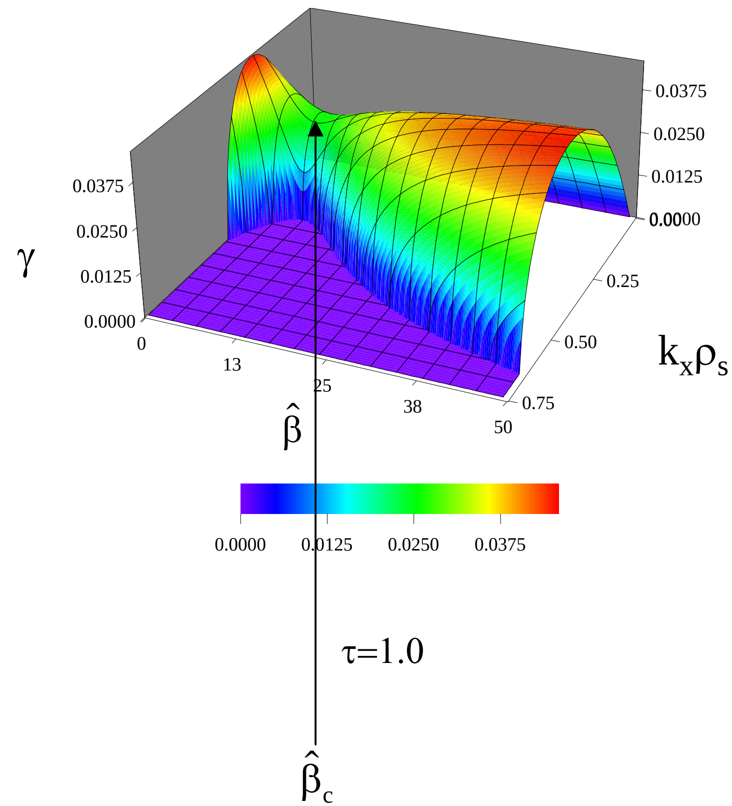




GROWTH RATE FOR ZONAL FLOW/FIELD
$k_y\rho_s=0.25,\ e\phi/T_e=\rho_s/L_n$



$\tau=0.0$



$\tau=1.0$

THRESHOLD CONDITION FOR L-H TRANSITION

MAXIMUM OF $(k_y\rho_s)^2\hat{\beta}_c$ FOR $k_y\rho_s$, τ AND $e\phi/T_e$

GROWTH RATE FOR ZONAL FLOW/FIELD

Dispersion Relation for the growth of zonal flow/field

$$\omega^2 = \left[\underbrace{\frac{1}{2}\frac{d^2\omega_0}{dk_x^2}k_x^2}_{Dispersion} \right]^2 + \frac{\Gamma^2}{2\omega_0}\frac{d^2\omega_0}{dk_x^2}k_x^2 \left[\underbrace{M_A\left(1 - \frac{\omega_0^2}{k_\parallel^2 v_A^2}\right)}_{Zonal\ flow} + \underbrace{M_B}_{Zonal\ field}\frac{\omega_0^2}{k_\parallel^2 v_A^2} \right] |\phi_0|^2$$

Reynold's Stress

Maxwell's Stress

Generalized Lighthill Criterion for modulational instability

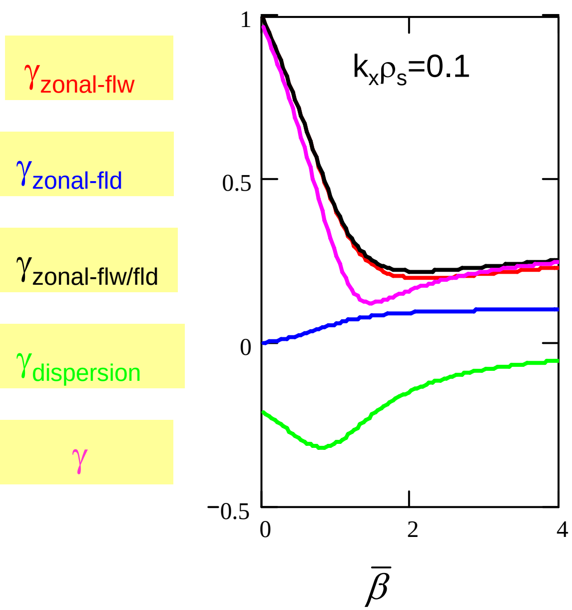
$$\frac{1}{\omega_0}\frac{d^2\omega_0}{dk_x^2}\left[M_A\left(1 - \frac{\omega_0^2}{k_\parallel^2 v_A^2}\right) + M_B\frac{\omega_0^2}{k_\parallel^2 v_A^2} \right] < 0$$

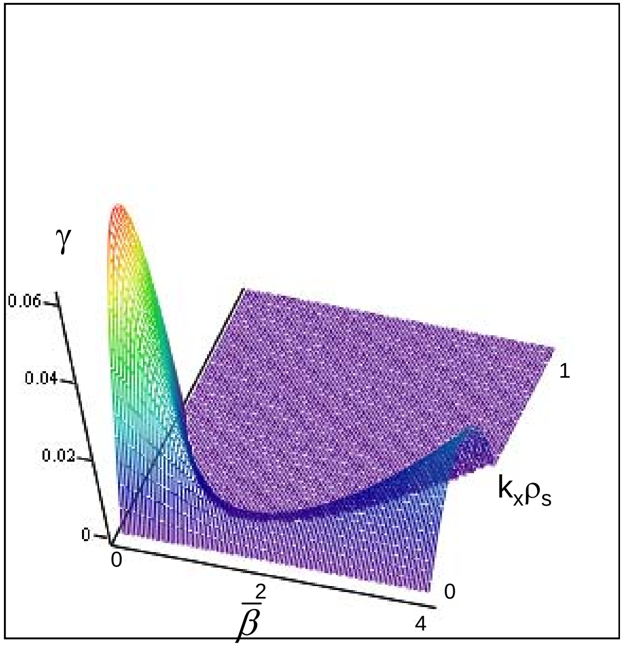
Four finite β effects: (1) modification of dispersion
(2) modification of matrix element M_A (from side-band modes)
(3) Maxwell Stress
(4) Zonal field

GROWTH RATE FOR ZONAL FLOW/FIELD

Parameters: $k_y\rho_s=0.25$, $\tau=0$, $e\phi_0/T_e=\rho_s/L_n$

Rogers and Drake, PRL 79,229 (1997)





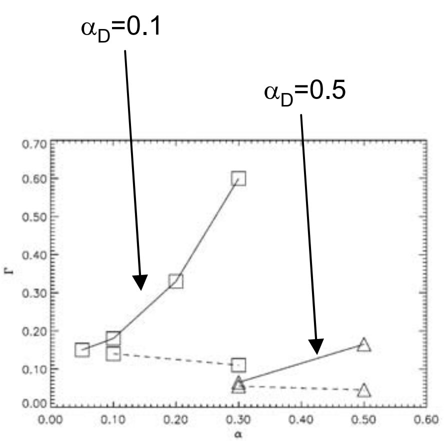


FIG. 3. Normalized particle flux Γ vs α.

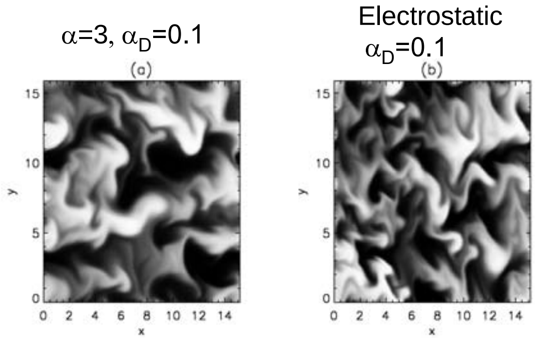
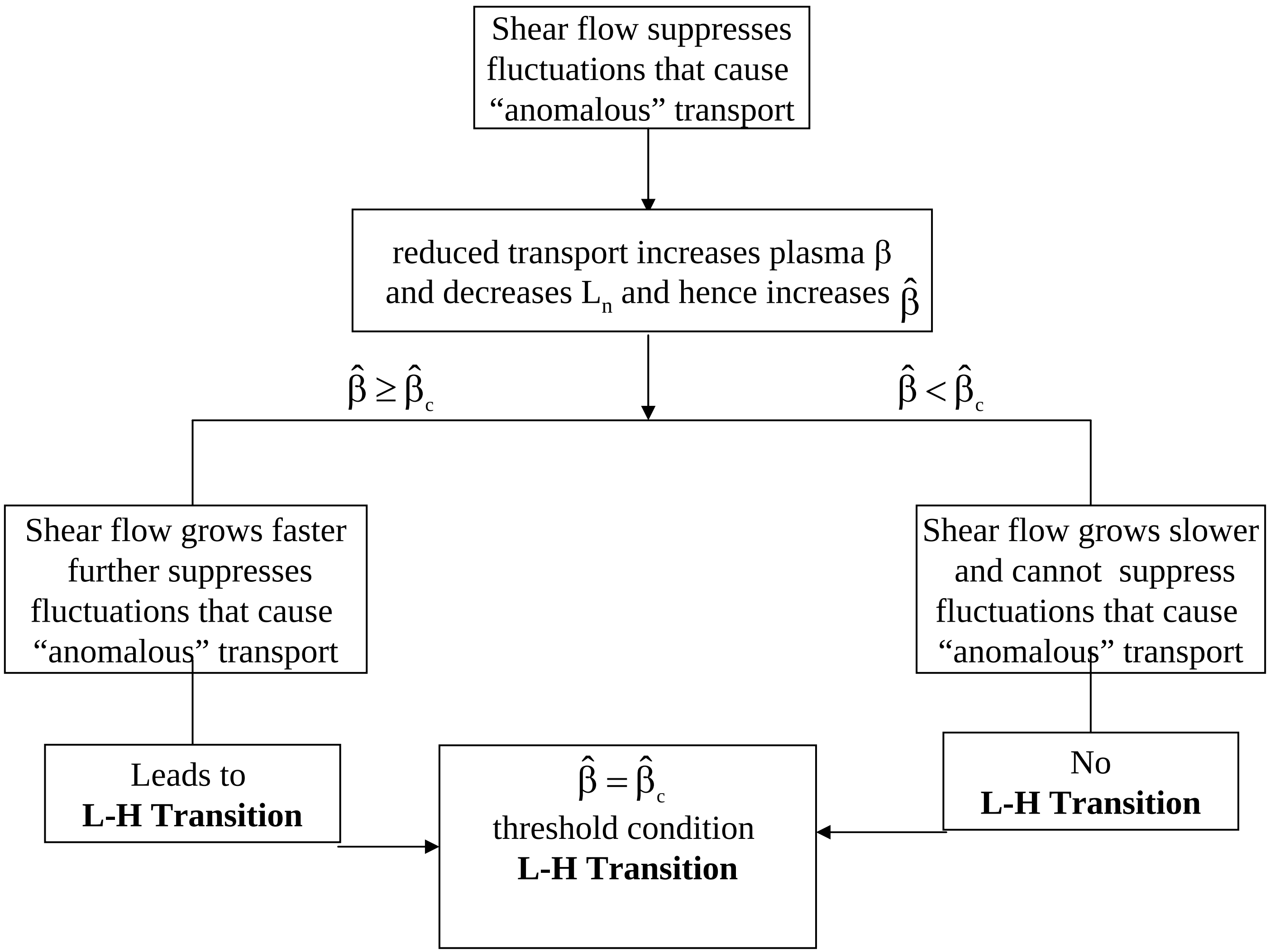


FIG. 4. Density perturbations: (a) electromagnetic, (b) electrostatic.

For $\tau=1$, dominant drive is zonal-flow $\gamma_{zonal\text{-}flow}$

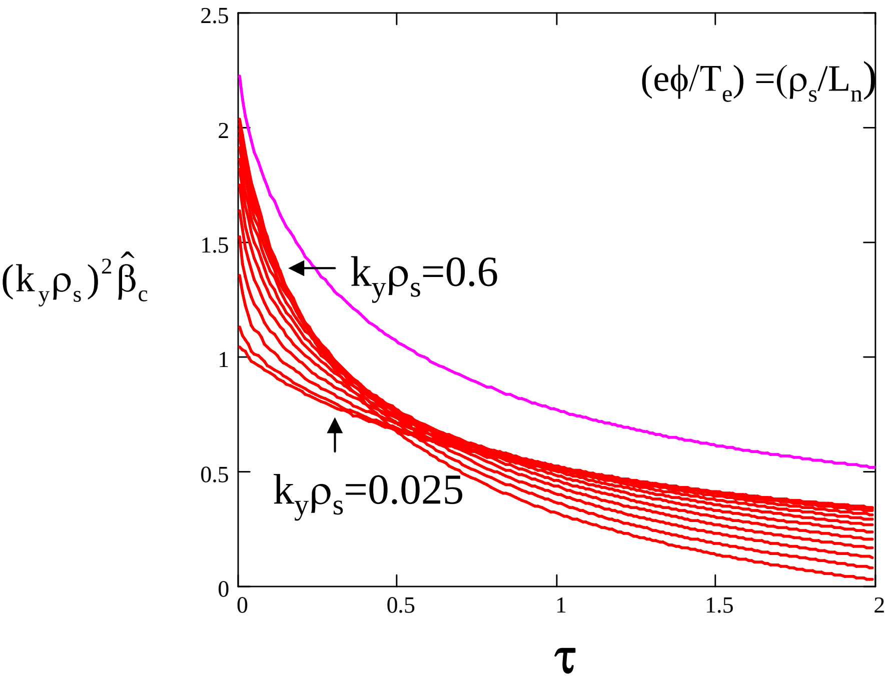
SCENARIO FOR L-H TRANSITION

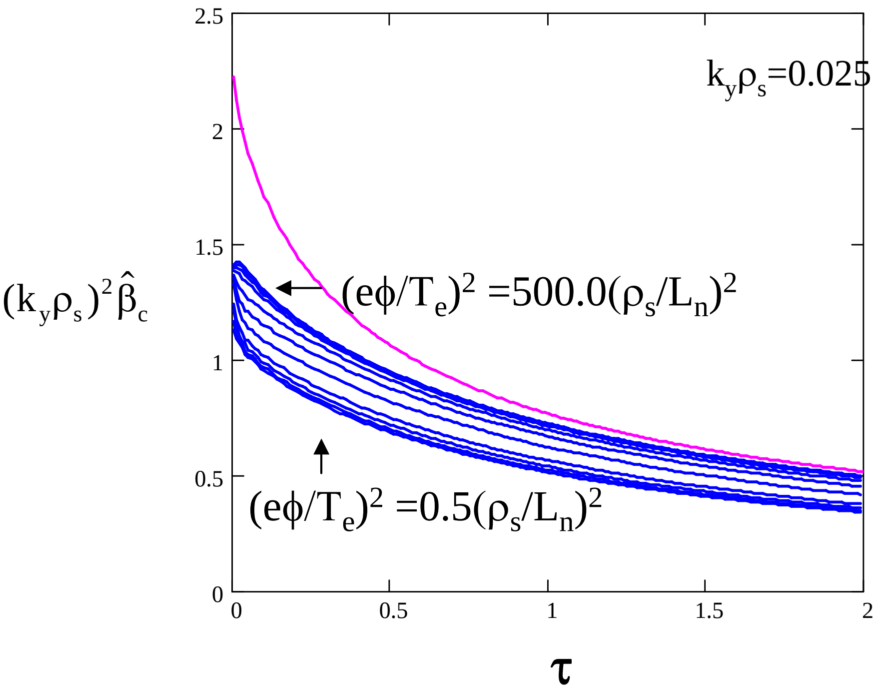


$(k_y \rho_s)^2 \hat{\beta}_c$ vs τ

$k_y\rho_s$=0.025, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6

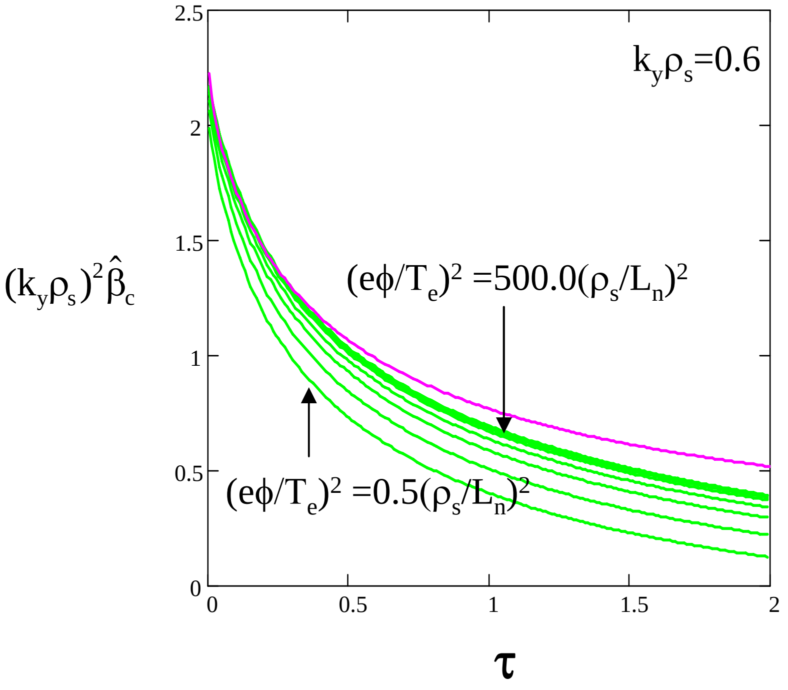
$(e\phi/T_e)^2$ =(500.0, 200.0, 100.0, 50.0, 20.0, 10.0, 5.0, 2.0, 1.0, 0.5)$(\rho_s/L_n)^2$

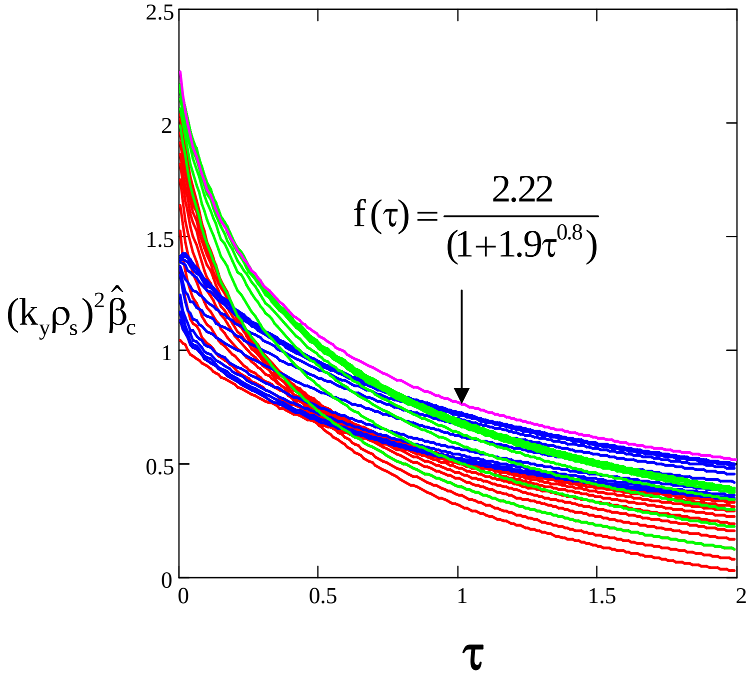




$(k_y\rho_s)^2\hat{\beta}_c$ vs τ

$(e\phi/T_e)^2$ =(500.0, 200.0, 100.0, 50.0, 20.0, 10.0, 5.0, 2.0, 1.0, 0.5)$(\rho_s/L_n)^2$





CRITICAL PARAMETER Λ OR CRITICAL T_E

$$k_y^2 \rho_s^2 \hat{\beta} = 2.22/(1+1.9\tau^{0.8})$$

$$k_y = 2\pi/L_0, \quad L_0 = 2\pi q\left(\frac{\nu_{ei} R \rho_s}{2\Omega_e}\right)^{1/2}\left(\frac{2R}{L_n}\right)^{1/4}$$

$$\frac{\rho_s^2}{L_0^2}\left(\frac{4\pi n T_e}{B^2}\right)\frac{q^2 R^2 (1+1.9\tau^{0.8})}{L_n^2} = \frac{1.11}{2\pi^2}$$

$$T_{ec}(\text{keV}) = 0.46\left(\frac{B^2(\text{T}) Z_{eff}}{1+1.9\tau^{0.8}}\right)^{1/3}\left(\frac{1}{R(\text{m}) A_i}\right)^{1/6} L_n^{1/2}(\text{m})$$

OR

$$\Lambda = \frac{T_e(\text{keV})\left(R(\text{m}) A_i\right)^{1/6}(1+1.9\tau^{0.8})^{1/3}}{\left(B^2(\text{T}) Z_{eff}\right)^{1/3} L_n^{1/2}(\text{m})}, \quad \text{with } \Lambda_c = 0.46$$

DATA FROM DIII-D
R. GROEBNER AND P. GOHIL

SHOT SERIES :	078151, 078153, 078155, 078156	I_p scan (1.0-2.0 MA) B_T=2.1 T, n_e=4.0x10^{19}/m^3
	078161, 078165, 078167, 078169	B_T scan (1.1-2.1 T) I_p=1.0 MA, n_e=4.0x10^{19}/m^3
	084026, 084032, 084040, 084044	n_e scan (1.4x10^{19}-3.9x10^{19} /m^3) I_p=1.3 MA, B_T=2.1T
	08830, 089348	Random selection B_T=2.1 T I_p=1.0 MA
	102014, 102015, 102016, 102017 102025, 102026, 102029	
SHOTS	96338, 96348	different ∇B drift I_p=0.97 MA, B_T=2.1 T
	99559, 100162	

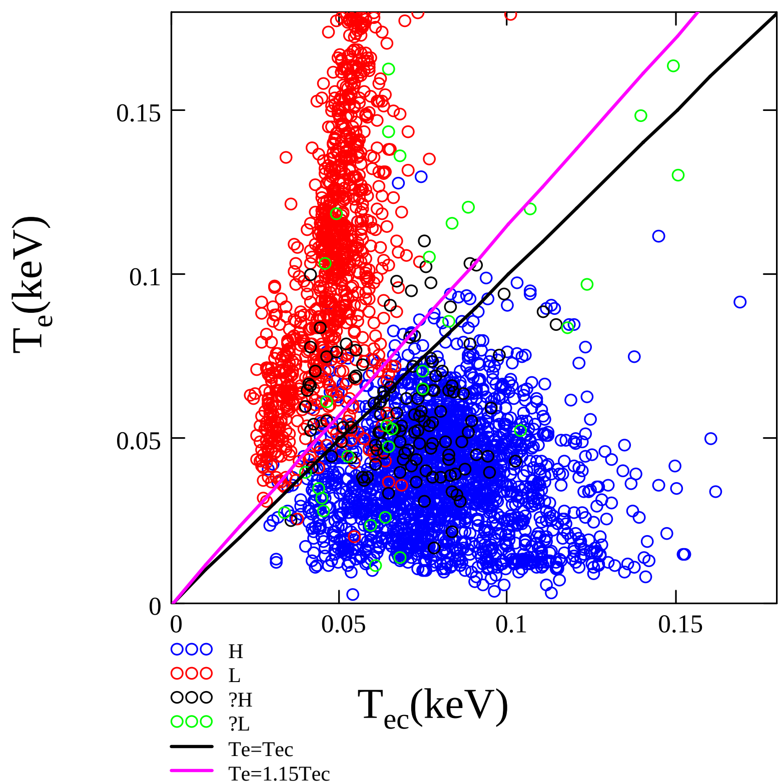
Data T_e and L_n back-averaged over three points

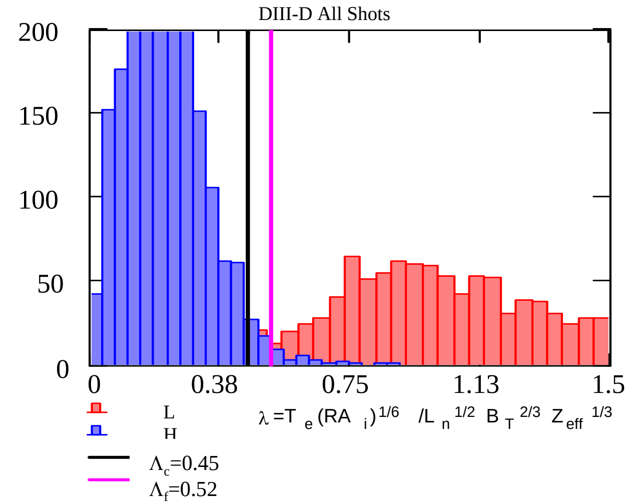
To remove fluctuations from “turbulence”

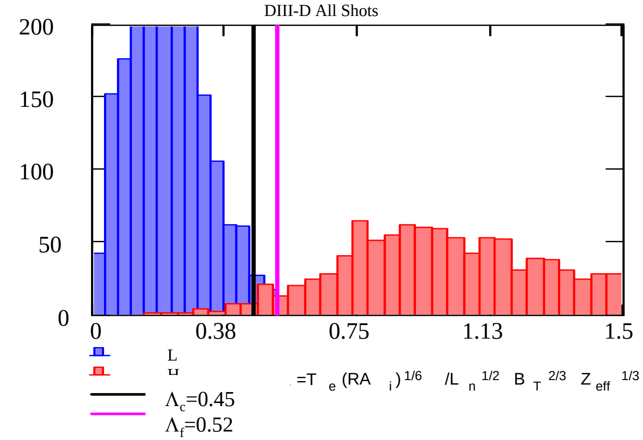
COMPARISON WITH DIII-D
(IN COLLABORATION WITH R. GROEBNER)--2

$$T_{ec}(\mathrm{keV}) = 0.45\frac{[B_T(T)]^{2/3}[L_n(m)]^{1/2} Z_{eff}^{1/3}}{[R(m)A_i]^{1/6}}$$

$$\Lambda = \frac{T_e(\mathrm{keV})\left(R(m)A_i\right)^{1/6}}{[B_T(T)]^{2/3}[L_n(m)]^{1/2} Z_{eff}^{1/3}}$$





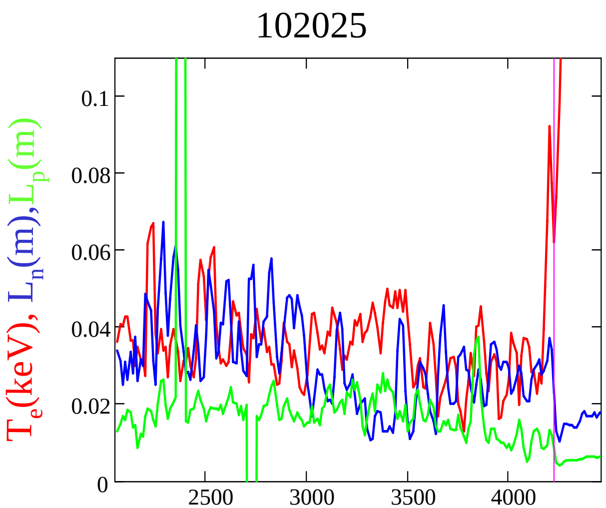


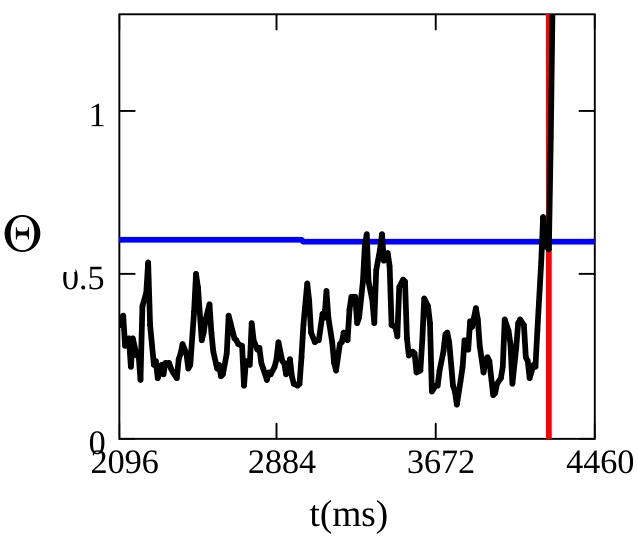
COMPARISON WITH DIII-D

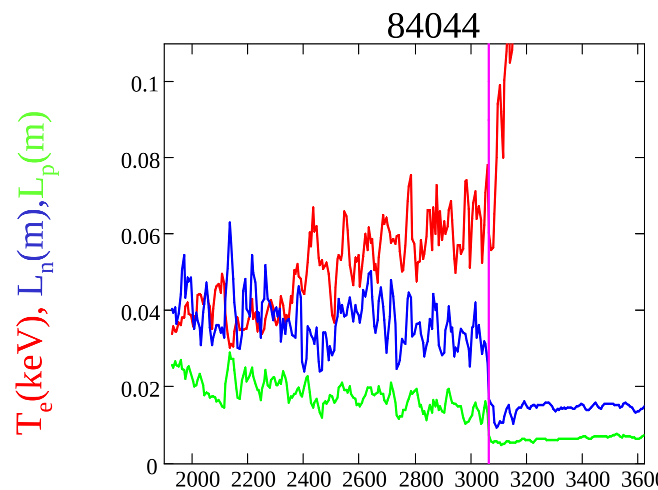
B_T=2.07T, I_P=1.57 MA, <n>=4.7x10^{19} m^{-3},
∇B towards X point till t=3450 ms, changed to away

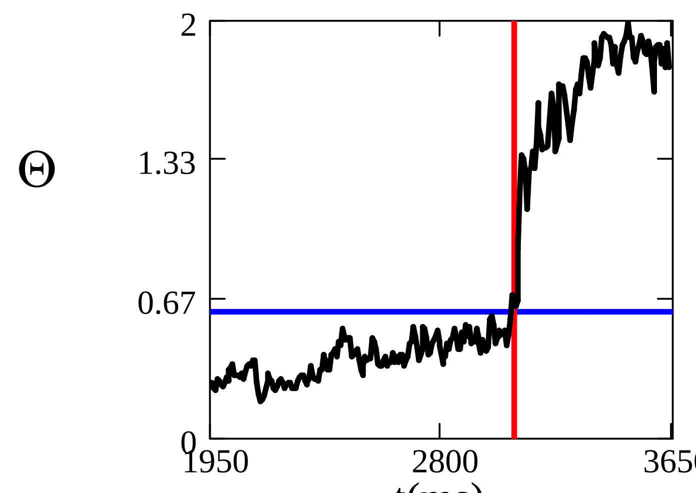
B_T=2.11T, I_P=1.33 MA, <n>=3.7x10^{19} m^{-3},
∇B towards X point

$$\Theta = \frac{T_e(\text{keV})}{\sqrt{L_n(\text{m})}}$$





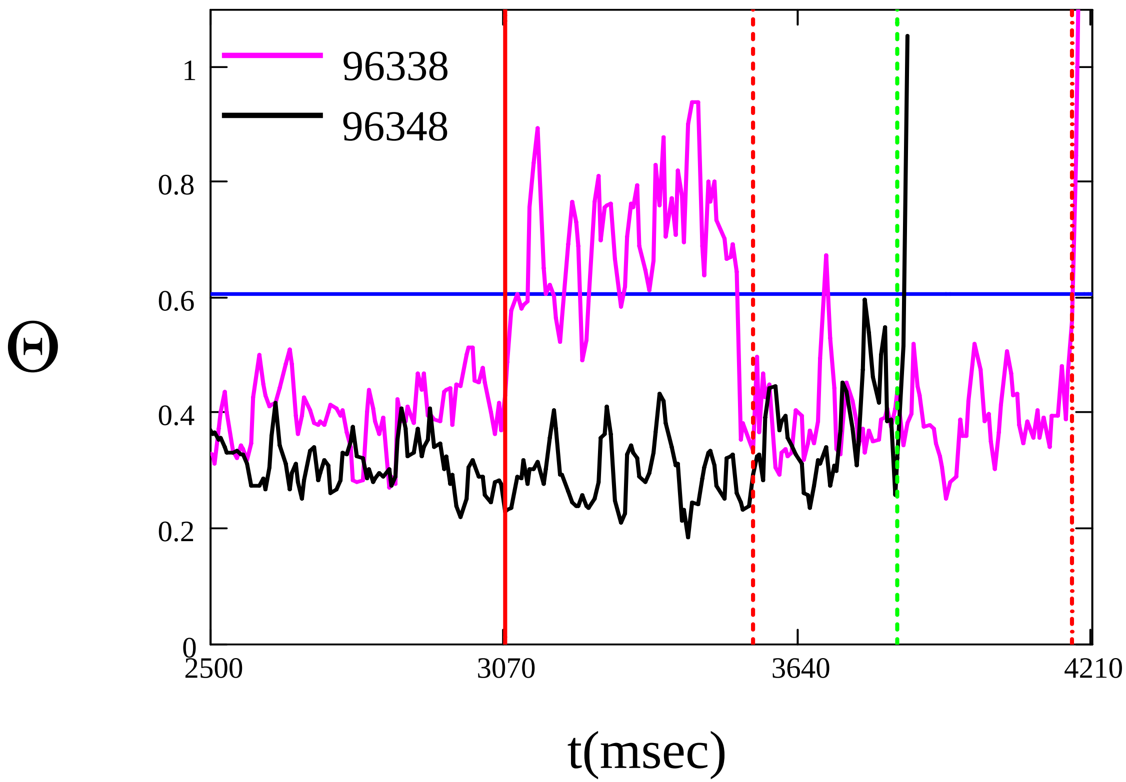




COMPARISON WITH DIII-D
GRAD B TOWARDS AND AWAY FROM THE X POINT

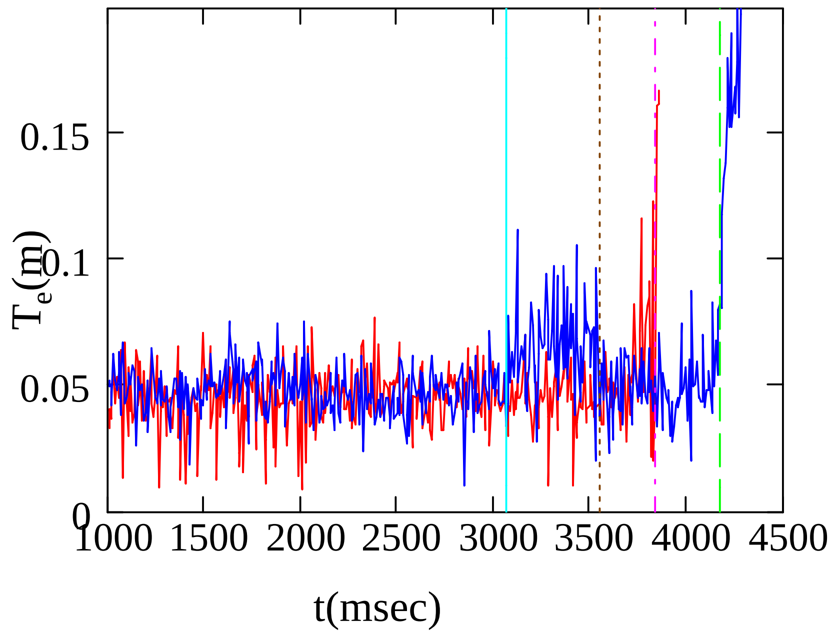
I_p=0.97 MA, B_T=2.12
Shot #96338 (∇B drift to X point)
Shot #96348 (∇B drift away X point)

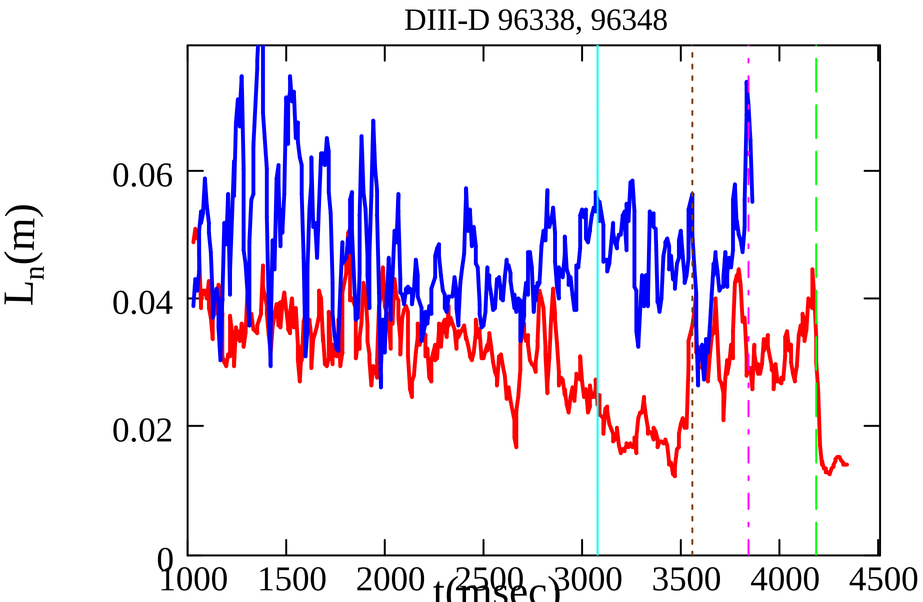
$$\Theta = \frac{T_e(\text{keV})}{\sqrt{L_n(\text{m})}}$$



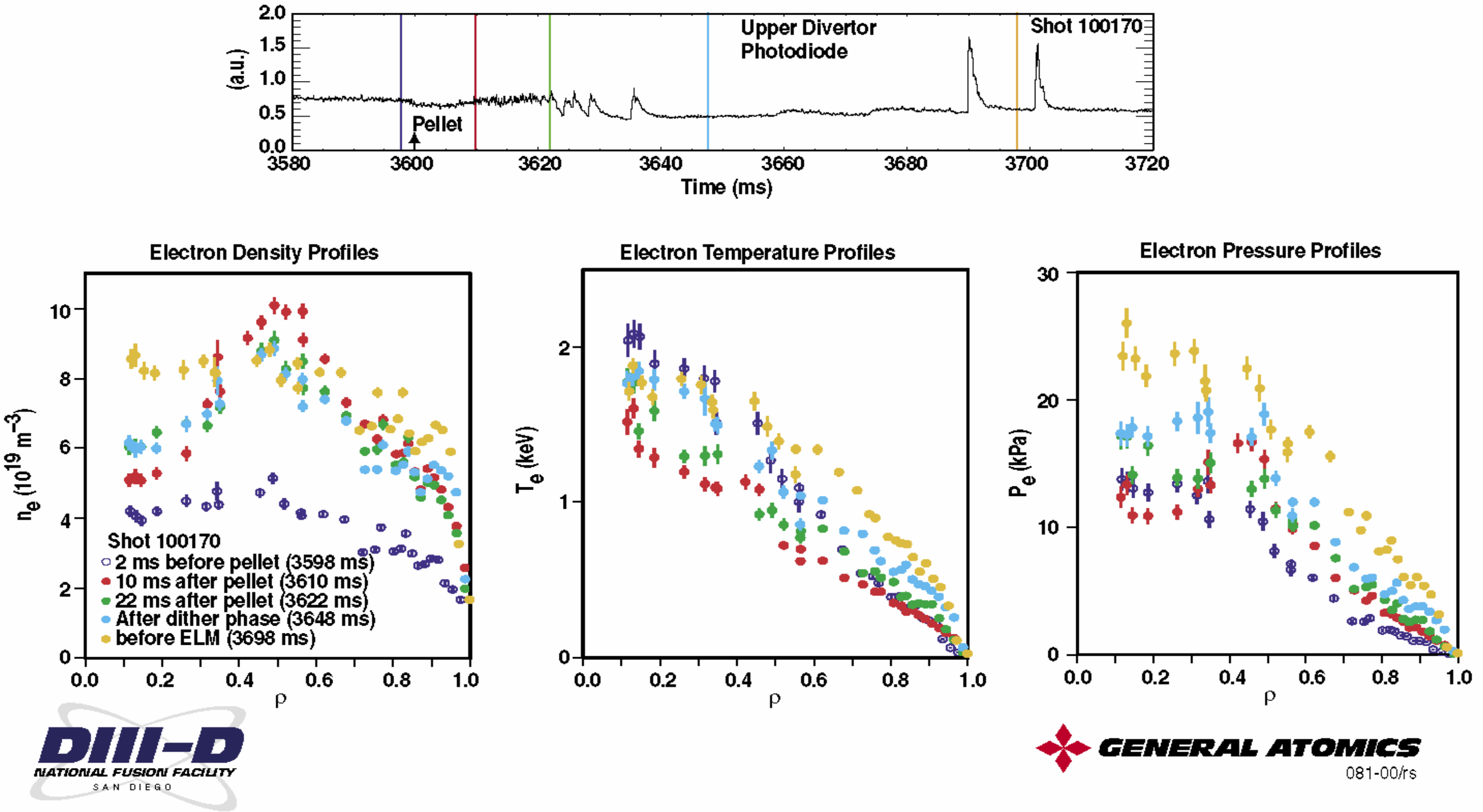
COMPARISON WITH DIII-D
GRAD B TOWARDS AND AWAY FROM X POINT

Shot #96338 (∇B drift to X point)
Shot #96348 (∇B drift away X point)





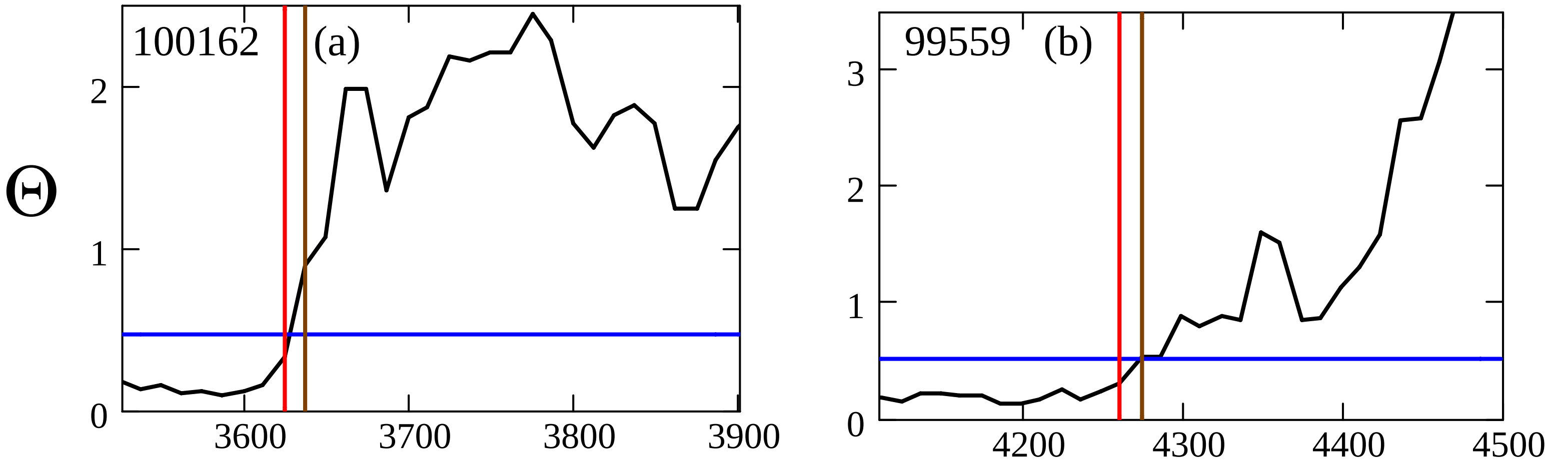
GOHIL ET AL. PHY. REV LETT. 86, 644 (2001)



Strong reduction in L_n in the edge 2 msec prior to transition

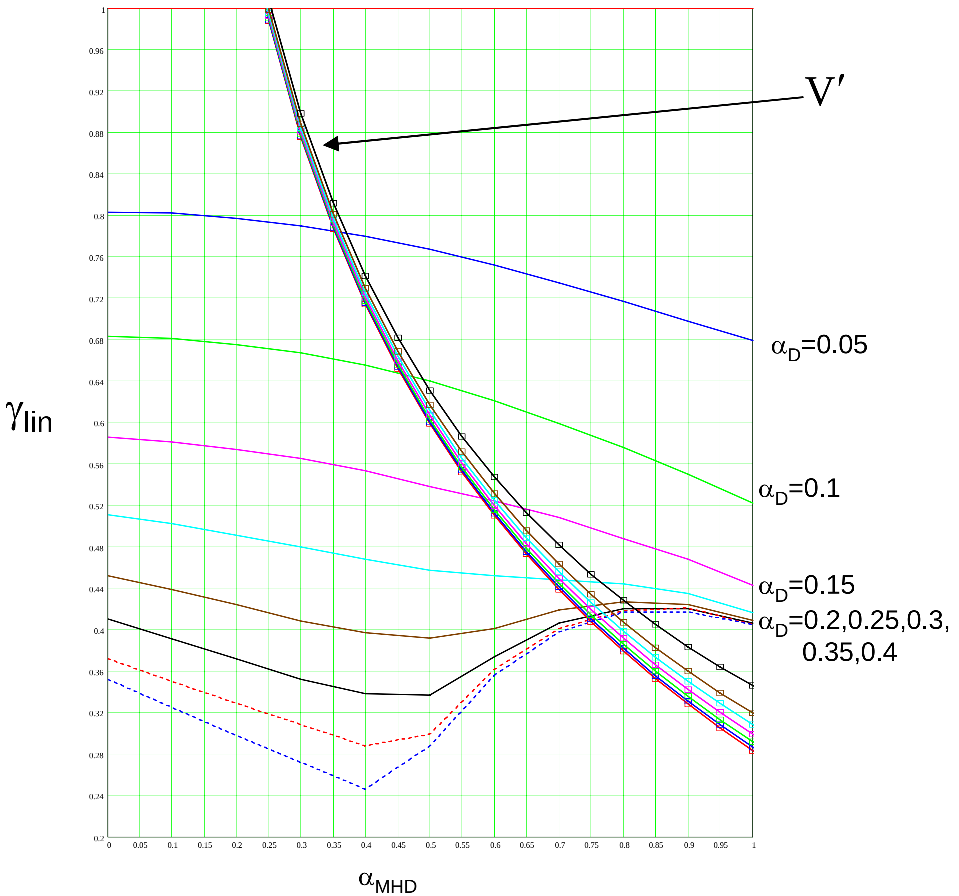
COMPARISON WITH DIII-D
WITH PELLET SHOTS 100162, 99559

Pellet Induced H mode

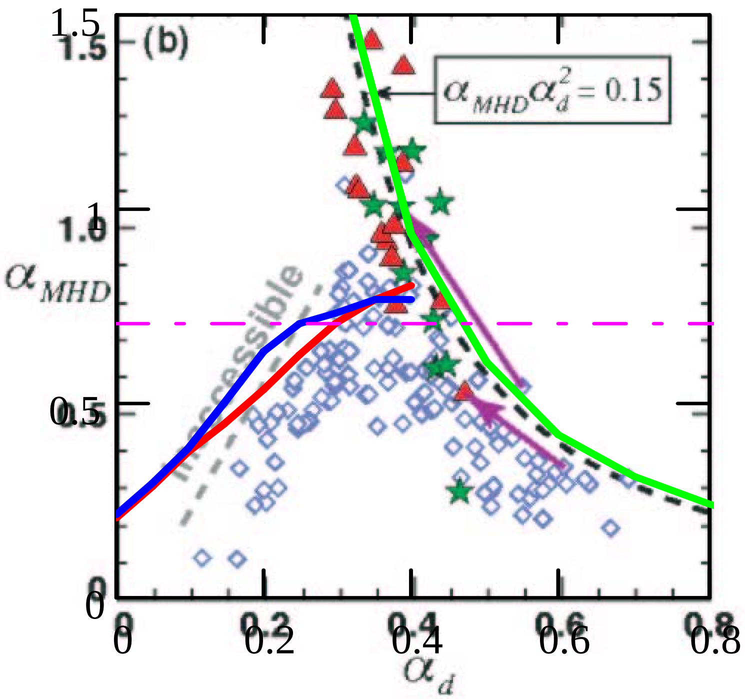


STABILITY OF ZONAL FLOWS

- Zonal flow responsible for saturation of finite β drift-wave turbulence
- Level of zonal flow determined by condition $V^{/} = \gamma_{linear}$
- If zonal flow unstable for amplitudes BELOW stability criterion, turbulence not saturated => very large edge transport => leads to disruption (Greenwald limit)



LaBombard et al NF 2005



CONCLUSIONS AND FUTURE WORK-I

•Derived generalized dispersion relation for shear/zonal flow with finite beta effects.

•Theory gives reasonable preliminary agreement with simulations and data on DIII-D and C-MOD

Future work

• undertake more careful study of the nonlocal dispersion relation

• study the low-dimensional nonlinear equations with finite beta to understand nature of the H mode attractor
  - -fixed point (non-ELMing),
  - -chaotic (ELMing)

• current theory is in terms of plasma parameters (T_e, T_i L_n) and not in terms of true control parameters. Use this simple expression in 1D and 2D predictive transport codes To obtain threshold power dependence and then make predictions for future devices

• Extend theory to core enhanced confinement modes

CONCLUSIONS AND FUTURE WORK-II

- Incorporate shaping effects in model for threshold for KH instability of zonal flows and finite beta drift wave/DRBM modes

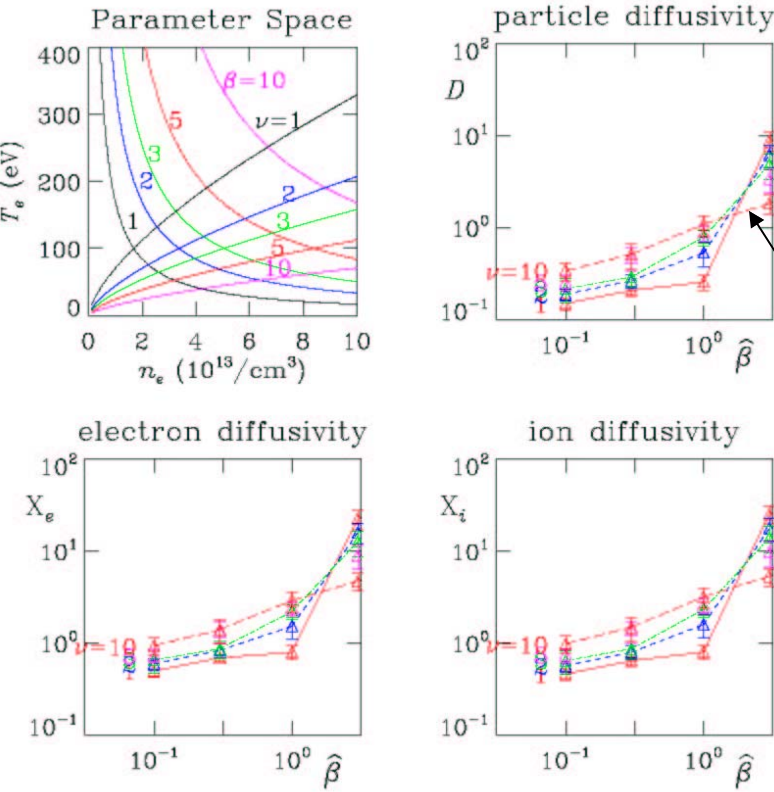
- Explore the “interaction” region between the stability boundary for KH and LH to identify type of ELMS. ELMS are interplay between ballooning type modes and zonal and/or shear flow

- Extend study to full two dimensional eigenvalue stability of modes with shear to determine stability boundary more accurately

- Explore the connection of the “density-limit” boundary in (α_{MHD}, α_D) space to the Greenwald limit

COMPARISONS

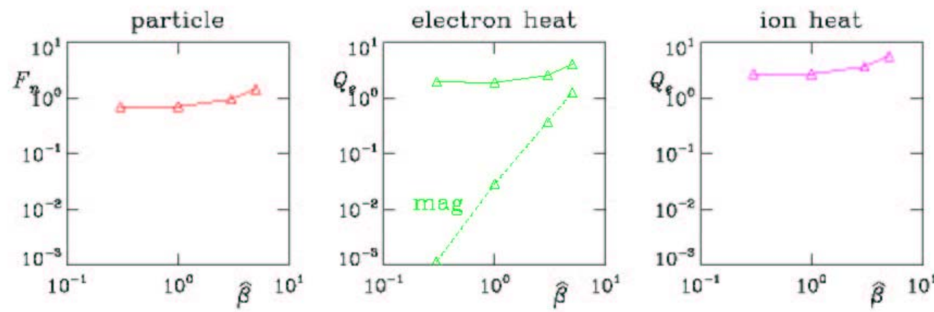
Scott NJP, 4, 52.1-52.30, (2002)
Braginskii Equations



$L_p = 4.25$ cm, R=169 cm, $L_n = 2L_T$

$\alpha_{MHD} = 0.1\hat{\beta} \qquad \alpha_D = 0.4\nu^{-1/2}$

Scott IAEA-11-S7, (2005)
Gyrokinetic Edge Code



$$\frac{R}{L_T} = 30,\ L_n = 2L_T,\ L_p = \frac{2}{3}L_T$$

$$\alpha_{MHD} = 0.09\hat{\beta} \qquad \alpha_D = 0.46$$

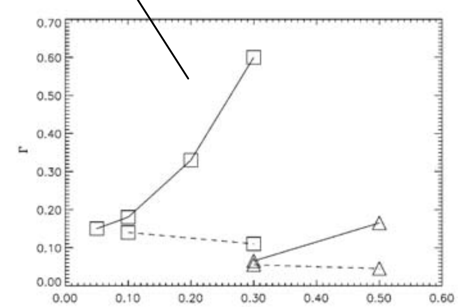


FIG. 3. Normalized particle flux Γ vs α.