Role of zonal flows and magnetic fields in turbulence regulation and the formation of transport barrier

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L-mode

- **Anomalous transport due to microscale turbulence**

$$D_T \sim \chi_T \sim \nu_T \sim \frac{\gamma}{k_\perp^2}$$

– **Particle flux** $\langle n v_x \rangle = -D_T \partial_x \langle n \rangle$

– **Heat flux** $\langle T v_x \rangle = -\chi_T \partial_x \langle T \rangle$

– **Momentum flux** $\langle v_x v_y \rangle = -\nu_T \partial_x \langle U_y \rangle$

Here, D_T**,** χ_T**, and** ν_T **are transport coefficients [turbulent particle and heat diffusivities, and eddy viscosity]**

- **cf:** $D_T \sim \chi_T \sim \nu_T \sim vl \sim \tau_c l^2$

L-H transition

Stabilization of turbulence and reduction in transport

$\Leftrightarrow$ **How D_T, χ_T, and ν_T are affected by $\mathrm{E} \times \mathrm{B}$ shear flows and magnetic fields??**

$\Leftrightarrow$ γ, $k_\perp$ (v, l, τ_c) **are dynamical quantities!**

NOTE: intermittent transport due to coherent structures (e.g. streamers, blobs, etc) $\Rightarrow$ PDFs are required (e.g. Kim & Diamond PRL 03)

Role of $\mathrm{E} \times \mathrm{B}$ shear flows on transition

RZFs trigger transition while mean flows maintains H-mode [Kim & Diamond PRL 03]

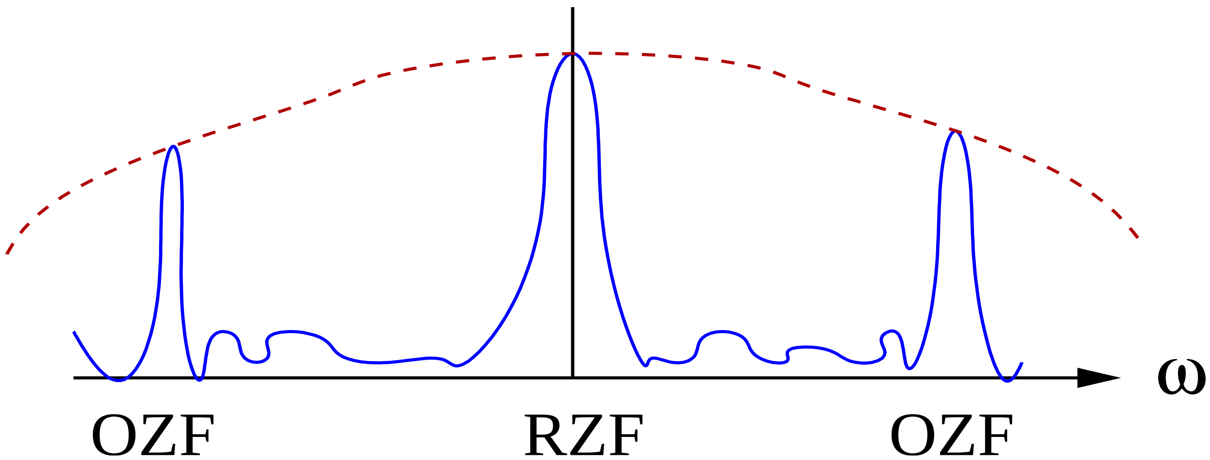
- **Mean flow (coherent shearing)**

$$\langle V_E \rangle = \langle V_\theta \rangle - \frac{B_\theta}{B} \langle V_\phi \rangle - \frac{1}{eB_z n} \frac{\partial p_i}{\partial r} + \tau$$

- **Zonal flows (ZFs)**
  - **Random ZFs (RZFs): low-frequency ($\omega_z \sim 0, \Delta\omega_z \sim 5$kH)**

$$\partial_t \phi_{ZF} = \langle \tilde{v}_x \tilde{v}_y \rangle - \nu \phi_{ZF}$$

  - **Oscillatory ZFs (OZFs): high-frequency**



- **Quantitative prediction for D_T, χ_T and ν_T due to these shear flows**
  - **Scalings of flux Γ with shearing rate [$\Gamma \propto \Omega^{-\alpha}$ with $2 \lesssim \alpha \lesssim 3.6$ (Boedo et al 02)]**
  - **Model dependence of results: stronger reduction in dynamical models [Kim et al 03,04,05]**
  - **Reduction in transport vs fluctuation levels: $\Gamma = \langle n v_x \rangle = \sqrt{\langle n^2 \rangle}\sqrt{\langle v_x^2 \rangle}\cos\delta_n$ (cross-phase $\cos\delta_n$)**

Effects of magnetic fluctuations for finite β plasmas

- Generation of zonal fields [Guzdar et al 01, Gruzinov et al 03]
- Alfvenization $\Leftrightarrow$ increase in the memory time τ_c

$\Rightarrow$ Quench transport but not necessarily fluctuation levels

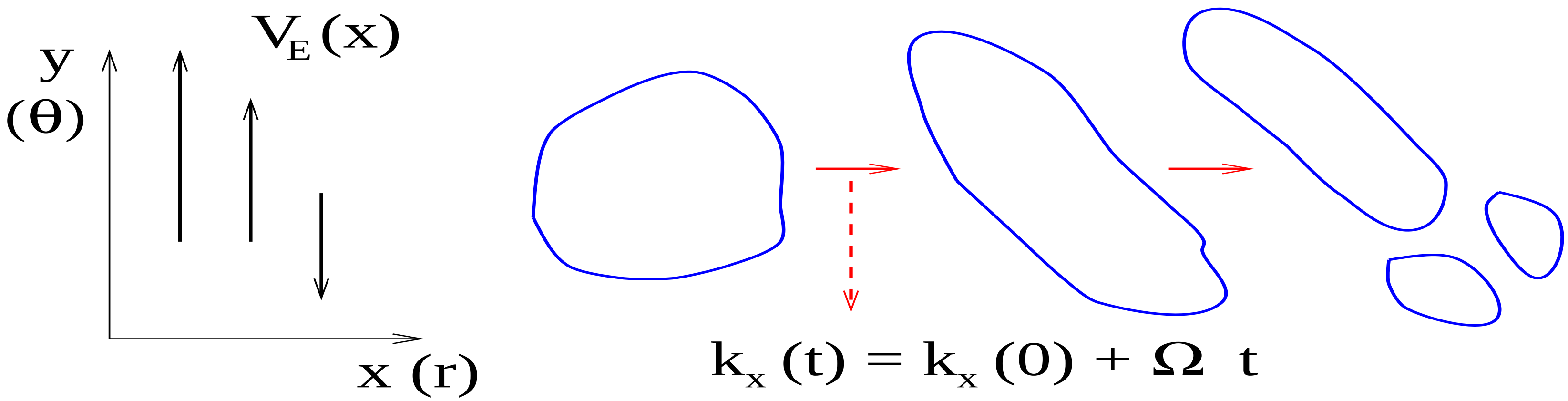
$\Rightarrow$ Reduction in the growth of RZFs? $\langle v_x v_y - b_x b_y \rangle = -\nu_T \langle U_y \rangle$

$\Rightarrow$ Reduction in χ_T, D_T vs ν_T? $\Leftrightarrow$ disparate transport of particles and momentum

Outline

1. **Effects of shear flows on fluctuation levels (enhanced dissipation)**
2. **Effects of shearing on transport (passive scalar field model with OZFs)**
3. **Effects of magnetic fields in 3D RMHD**
4. **Conclusions/discussion**

I. Enhanced dissipation due to shearing



- **Rapid generation of fine scales due to eddy distortion by shear flows**

$$k_x(t) = k_x(0) + k_y \int^t \Omega(t')dt' \qquad [\mathbf{U} = -x\Omega(t)\hat{y}]$$

$\Rightarrow$ **Reduction in fluctuation levels**

1. **Coherent shearing with constant Ω ($k_x^2 \propto t^2$)**

$$Q = D \int^t dt' k_x^2(t') \propto D k_y^2 \Omega^2 t^3, \quad \tau_\Delta = (\tau_\eta / \Omega^2)^{1/3} \qquad [\tau_\eta = 1/D k_y^2]$$

2. **Random shearing by RZFs (finite τ_{ZF}, $k_x^2 \propto t$)**

$$Q \propto D k_y^2 \tau_{ZF} \Omega_{rms}^2 t^2, \quad \tau_D = (\tau_\eta / \tau_{ZF} \Omega_{rms}^2)^{1/2} = (\tau_\eta / \Omega_{eff})^{1/2}$$

3. **Coherent shearing by OZFs ($\Omega(t) = -\Omega_m \sin \omega_z t$) for $\Omega_m \gg \omega_z$**

$$Q \propto D[k_y^2(1 + \Omega_m^2/{\omega_z}^2) + k_x^2]t, \quad \tau_* = \tau_\eta {\omega_z}^2 / \Omega_m^2$$

$$\Rightarrow \boxed{\tau_\Delta \lesssim \tau_D \lesssim \tau_*}$$ **for** $\Omega_m > \omega_z > \tau_{ZF}^{-1} > \tau_\eta^{-1}$

II. Transport of passive scalar fields

Transport $\Leftrightarrow$ Irreversibility (dissipation, stochasticity)

I. Wave dominated background ($\omega \gg \gamma$)

$\Leftrightarrow$ Dissipation, resonance/critical layers

- **Mean flow: resonance $\omega - U_0 k = 0$ with $D_T \propto \Omega$ [Kim & Diamond 03]**
- **RZFs: resonance broadening with $D_T \propto \Omega_{rms}$ [Kim & Diamond 04]**
- **OZFs: resonance $\omega - n\omega_z = 0$ (n integer) with $D_T \propto \Omega_m$ [Kim 06]**

II. Turbulence dominated background ($\omega \ll \gamma$) $\Leftrightarrow$ Stochasticity

$\Rightarrow$ No effect of mean shearing on transport (no time for shearing to act)

Transport of passive scalar field n with OZFs

[Kim PoP 06]

$$(\partial_t + \mathbf{u}\cdot\nabla)n = D\nabla^2 n$$

- **Quasi-linear analysis:** $\mathbf{u} = \mathbf{U} + \mathbf{v}, \quad n = n_0(x) + n'$
  - $\mathbf{v}$**: Given (prescribed) turbulent flow**
  - $U(x,t) = -x\Omega(t)$ **with** $\Omega(t) = \Omega_m \sin\omega_z t$ **[OZFs]**
- **Solve for fluctuation for a given** $\mathbf{v}$

$$(\partial_t - x\Omega\partial_y)n' = -v_x\partial_x n_0 + D\nabla^2 n'$$

Let

$$n'(\mathbf{x},t) = \frac{1}{(2\pi)^3}\int d^3k\tilde{n}(\mathbf{k},t)e^{i(k_x(t)x+k_yy+k_zz)}$$

where

$$k_x(t) = k_x(0) + k_y\int^t dt_1\Omega(t_1)$$

and similarly for $\mathbf{v}$

- **Compute** $\langle {n'}^2\rangle, \langle n'v_x\rangle = -D_T\partial_x n_0$ **by using**

$$\langle\tilde{v}_x(\mathbf{k}_1,t_1)\tilde{v}_x(\mathbf{k}_2,t_2)\rangle = (2\pi)^2\delta(\mathbf{k}_1+\mathbf{k}_2)\psi(\mathbf{k}_2)\int\frac{d\omega'}{\pi}\frac{\gamma e^{-i\omega'(t_2-t_1)}}{[(\omega'-\omega)^2+\gamma^2]}$$

where $\omega > \gamma$

$\underline{\tau_\eta \gg \Omega_m/{\omega_z}^2(k_y x)}$:

$$\langle\langle n' v_x\rangle\rangle_T \sim -\frac{\partial_x n_0}{(2\pi)^2}\int d^2k\psi(\mathbf{k})\sum_{n=-\infty}^{\infty} J_n^2(\beta)\frac{\gamma+\mu}{(-n\omega_z+\omega)^2+(\gamma+\mu)^2}$$

$$\langle\langle n'^2\rangle\rangle_T \sim \frac{(\partial_x n_0)^2}{(2\pi)^2}\int d^2k\psi(\mathbf{k})\sum_{n=-\infty}^{\infty} J_n^2(\beta)2\tau_*\frac{\gamma+\mu}{(-n\omega_z+\omega)^2+(\gamma+\mu)^2}$$

where $\beta = k_y x\Omega_m/\omega_z$, $\mu = Dk_1^2(1+\Omega_m^2/{\omega_z}^2)$, $\tau_\eta = 1/Dk^2$

For $\underline{\beta \gg 1}$:

$$\langle\langle n' v_x\rangle\rangle_T \propto \frac{1}{|k_y U_m|}\,, \langle\langle n'^2\rangle\rangle_T \propto \frac{\tau_*}{|k_y U_m|}\,,$$

where $\tau_* = \tau_\eta {\omega_z}^2/\Omega_m^2 < \tau_\eta$, $U_m = x\Omega_m$ **is the amplitude of** U

- $\omega \gg \omega_z$ **(resonance where $\omega = n\omega_z$):**

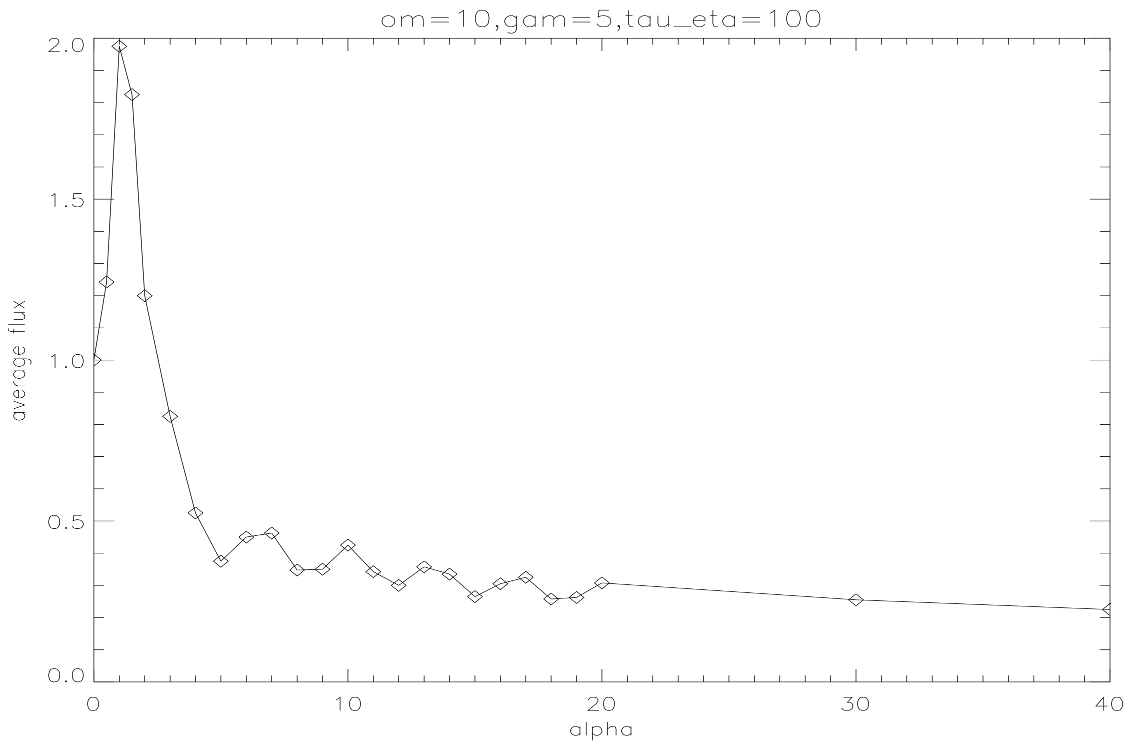


Figure 1: alpha$= \Omega_m/\omega_z, \omega/\omega_z = 10$

$\Rightarrow$ **OZFs reduce transport as Ω_m^{-1} for $\Omega_m \gg \omega_z$ ($\tau_* < \tau_\eta$)**

- $\omega \ll \omega_z$ **(no resonance):**

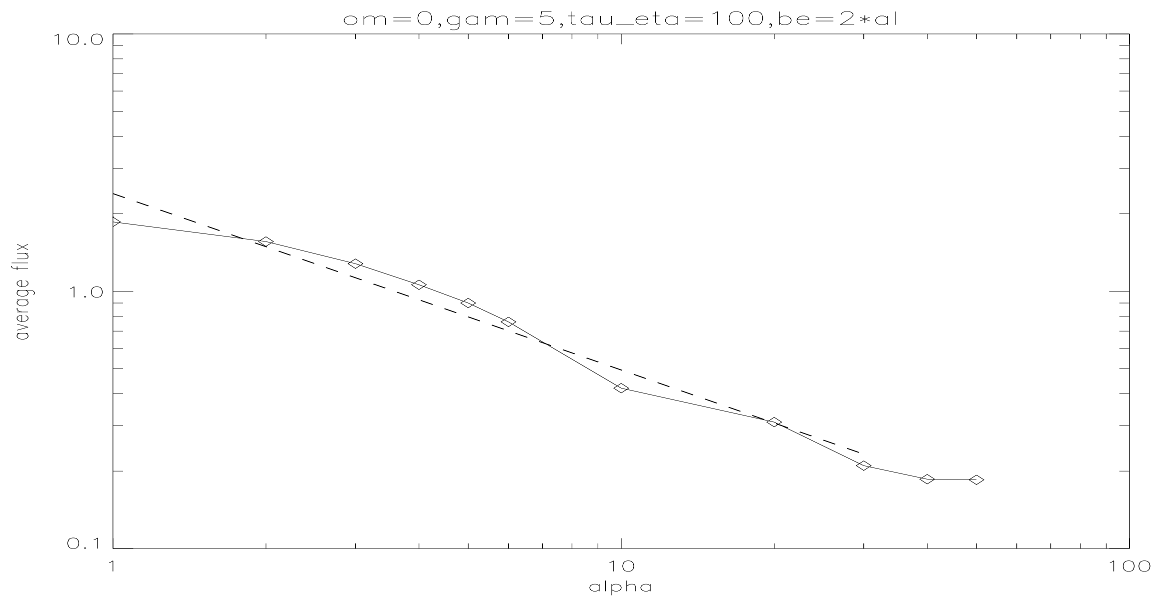
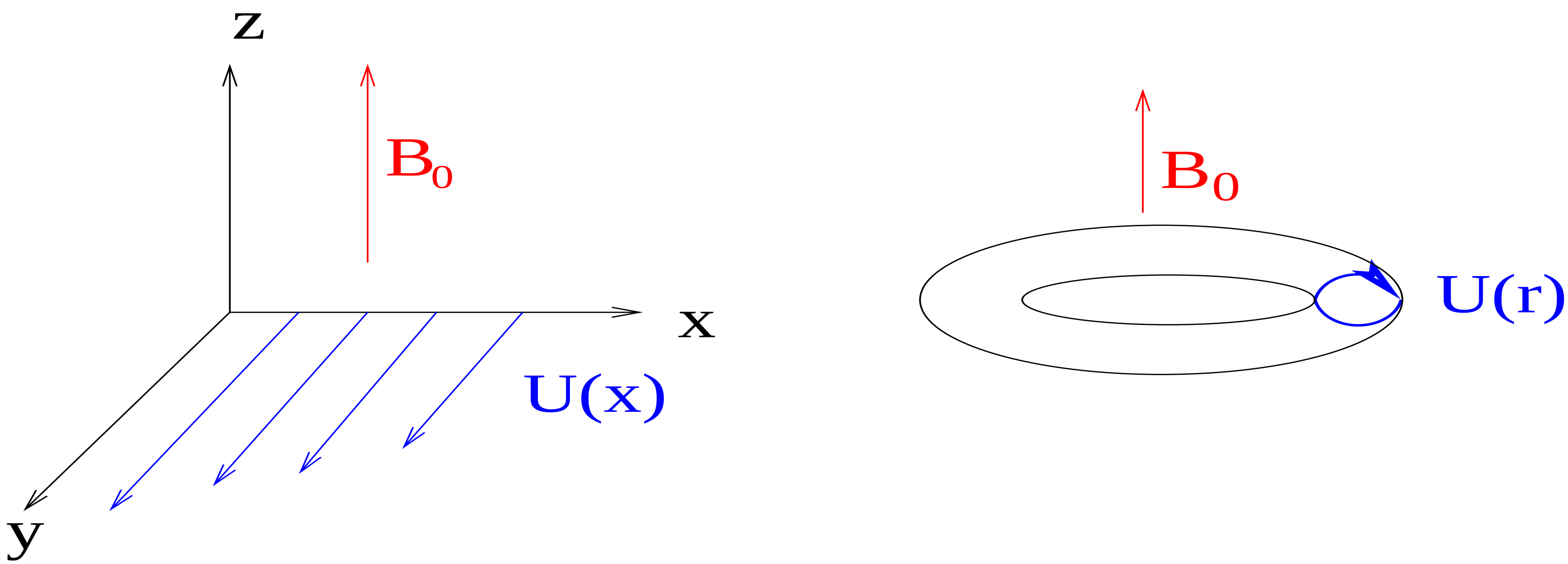


Figure 2: alpha$= \Omega_m/\omega_z, \omega/\omega_z = 0$

$\Rightarrow$ **OZFs reduce transport for** $\Omega_m < \omega_z$ **(**$\tau_* \sim \tau_\eta$**) [cf mean flow]**

III. 3D RMHD

[Kim, PRL 06]



• **Assume stationary mean shear flow $U(x)$ and turbulence dominated background with $\omega \ll \gamma$**

$$
\begin{aligned}
(\partial_t + \mathbf{u}\cdot\nabla)\omega &= -(\mathbf{B}\cdot\nabla)\,\nabla_\perp^2 a + \nu\nabla^2\omega + F \\
(\partial_t + \mathbf{u}\cdot\nabla)a &= B_0\partial_z\psi + \eta\nabla^2 a \\
(\partial_t + \mathbf{u}\cdot\nabla)n &= D\nabla^2 n\,.
\end{aligned}
$$

- $n = n_0 + n'$, $\mathbf{B} = B_0\hat{z} + \mathbf{b}'$, $\mathbf{U} = U(x)\hat{y} + \mathbf{u}'$,
- $\mathbf{b}' = \nabla\times a\hat{z} = (\partial_y a, -\partial_x a)$, $\omega = -\nabla_\perp^2\phi$, $\omega\hat{z} = \nabla_\perp\times\mathbf{u} = (\partial_x u'_y - \partial_y u'_x)\hat{z}$
- **Solve for $\hat{\omega}$, $\hat{a}$, and $\hat{n}$ in terms of forcings:**

$$
\langle\tilde{F}(\mathbf{k},t_1)\tilde{F}(\mathbf{k}',t_2)\rangle = \tau_f\delta(t_2 - t_1)\delta(\mathbf{k}+\mathbf{k}')\hat{\phi}(\mathbf{k})
$$

- **Compute ν_T and D_T via $\langle n'u'_x\rangle = -D_T\partial_x n_0$ and $\langle u'_x u'_y - b'_x b'_y\rangle = -\nu_T\partial_x U_0$**

In the limit of strong shear and magnetic fields:

$\xi \equiv \nu k^2/\Omega \ll 1$, $B_0 k_z/\Omega \gg 1$ **(**$\Omega = -\partial_x U_0$, $k = k_y$**)**

$$D_T \sim \nu \xi^{\frac{2}{3}} \frac{v^2}{\mu^2 B_0^2} < \nu_T\,, \qquad \nu_T \sim \nu \frac{v^2}{\mu^2 B_0^2}$$

$$\frac{\langle n'^2 \rangle}{(\partial_x n_0)^2} \sim \xi \frac{v^2}{k_z^2 B_0^2}\,, \qquad \langle u'^2_x \rangle \sim \frac{\xi v^2}{\mu^2}$$

where $v^2 = \tau_f \langle F^2 \rangle / \nu k^4$ **and** $\mu = k_z/k_H$

- B_0 **and** Ω **both reduce turbulent transport and amplitude in general**

- B_0 **does not reduce** $\langle u'^2 \rangle$

$\Rightarrow$ **More severe reduction in transport than amplitude for strong** B_0

$\Rightarrow$ **Normalized transport: Cross phase**

$$\cos\delta = \frac{\langle n' u'_x \rangle}{\sqrt{\langle n'^2 \rangle \langle u'^2_x \rangle}} \sim \left(\frac{Dk^2}{\Omega}\right)^{\frac{2}{3}} \left(\frac{\Omega}{B_0 k}\right) < 1$$

- **Disparity in** D_T **and** ν_T

$$\frac{D_T}{\nu_T} \sim \left(\frac{Dk^2}{\Omega}\right)^{\frac{2}{3}} \ll 1$$

$\Rightarrow$ **More efficient transport of momentum than particles!**

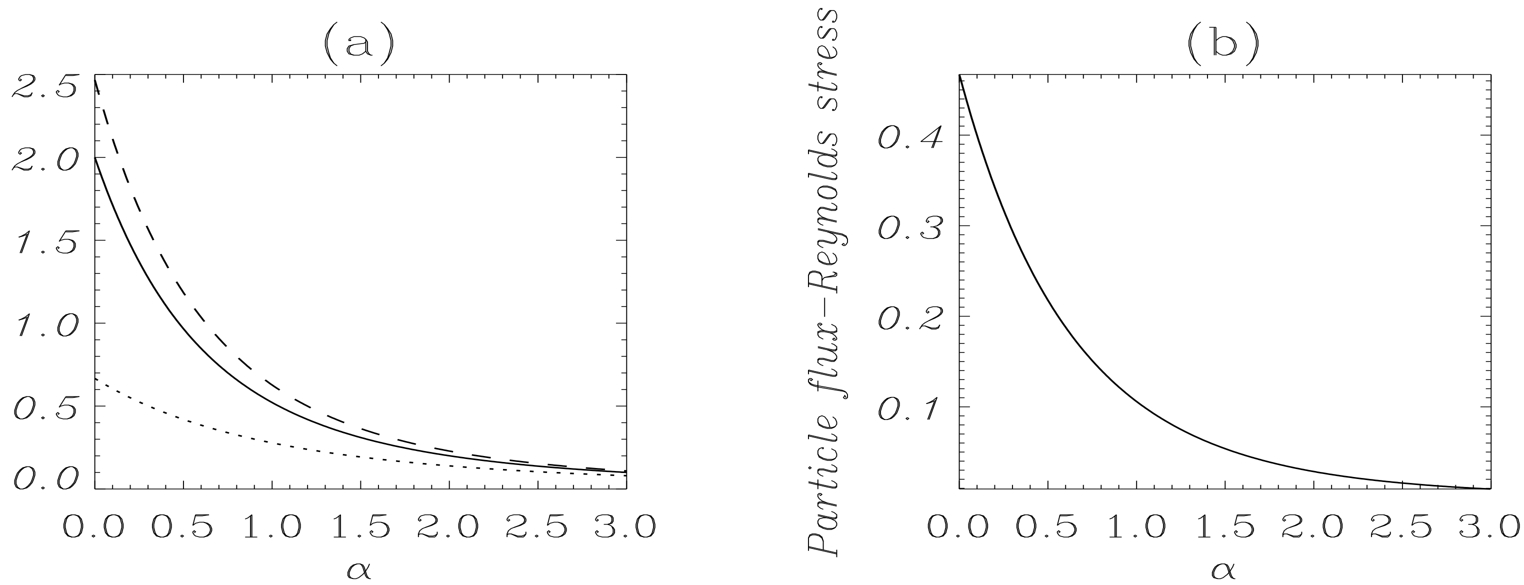
Stationary magnetic fields $D_t\hat{a} = 0$

EXACT solutions for $\xi = Dk^2/\Omega \ll 1$:

$$
\begin{aligned}
\nu_T &\sim -\xi\frac{v^2}{\Omega}\left[I_1(\alpha) - \left(\frac{B_0\mu}{\eta k}\right)^2 I_2(\alpha)\right] < 0 \\
D_T &\sim \xi\frac{v^2}{\Omega}I_3(\alpha) \\
\frac{\langle n'^2\rangle}{(\partial_x n_0)^2} &\sim \xi\frac{v^2}{\Omega^2}I_4(\alpha) \\
\langle {u'}_x^2\rangle &\sim \xi v^2 I_5(\alpha)
\end{aligned}
$$

Here, $v^2 = \tau_f\langle F^2\rangle/\nu k^4$**,** $\alpha = B_0^2\mu^2/\eta\Omega$**,** $\mu = k_z/k_H$**;** I_i**'s are monotonically decreasing functions of** α

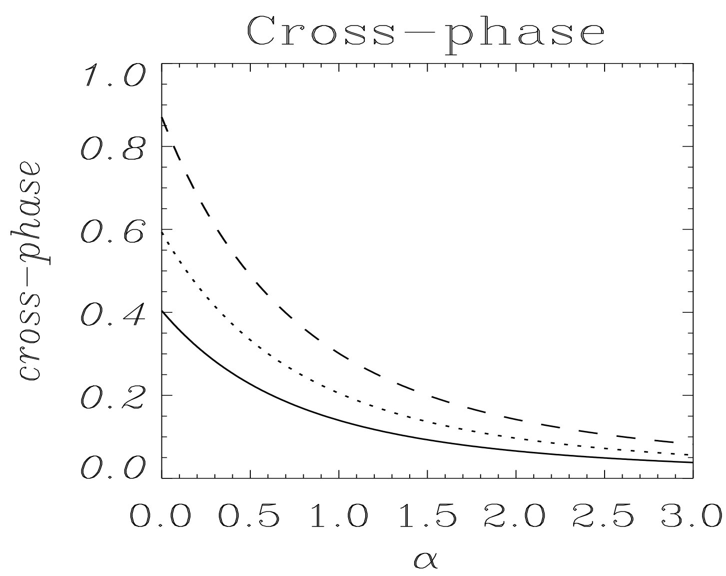
- **Comparison between ν_T (I_1, I_2) and D_T (I_3)**



$\Rightarrow$ **More reduction in D_T than ν_T**

$\Rightarrow$ **D_T can still be reduced in spite of the reduction in ν_T**

• **Effects of magnetic fields on cross-phase**



⇒ **Reduction in cross-phase due to magnetic fields**

⇒ **Possibility of significant reduction in transport without much reduction in fluctuation levels**

IV Conclusions

• **For strong shear $\Omega_m \gg \omega_z$, OZFS reduces transport as Ω_m^{-1} the effective decorrelation time $\tau_* = \tau_\eta {\omega_z}^2/\Omega_m^2 < \tau_\eta$**

$\Rightarrow$ **Turbulence regulation by OZFs less efficient than by RZFs?**

• **For $\omega \ll \gamma, \omega_z$, $\tau_* \sim \tau_\eta$; OZFs reduces transport even for $\Omega_m < \omega_z$**

• **For strong shear, $\langle n'^2 \rangle \propto \tau_{eff} \langle n' v_x \rangle$ for $\tau_{eff} = \tau_\Delta, \tau_D, \tau_*$**

• **Reduction in transport without much reduction in fluctuation levels due to magnetic fields**

• **Magnetic fields can facilitate barrier formation**

• **Origin and properties of forcings and their effects on transport?**

• **Transport in more realistic RMHD models with toroidal effects?**

- **Generation of zonal magnetic fields (dynamos)?**

- **Effects of magnetic fields (e.g. tearing modes) on particle/heat pinch?**