Linear Edge Stability and Toroidal Rotation

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Introduction

MHD model for ELMs:

- ELMs are MHD instabilities localized near the plasma edge
- low-n (toroidal mode number) peeling modes driven by edge current (bootstrap current), highly localized
- high-n ballooning modes driven by edge pressure gradient, more extended
- intermediate-n coupled peeling-ballooning modes (e.g. Snyder et al., PPCF **46**; 2004)

Effects of Sheared Toroidal Rotation:

- analytical theory (e.g. Connor et al., PPCF **46**, 2004; Furukawa et al., PPCF **46**, 2004) predicts stabilization of high-n ballooning modes in the s-α-model
- limitations: high n, large aspect ratio, no shaping
- 'Doppler shift': linear growth rate $\gamma \rightarrow \gamma + in\Omega(r)$ where $\Omega(r)$ is the rotation frequency
- discontinuity in the linear growth rate $\gamma(\Omega)$ for $n \rightarrow \infty$
- linear stability limits, most unstable mode number, mode extend and mode phase

Linear MHD Stability Analysis

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normalized MHD equations:

- *continuity equation*

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

- *induction equation*

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

- *momentum equation*

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla p + (\nabla \times \mathbf{B}) \times \mathbf{B}$$

- *energy equation*

$$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla T - (\gamma - 1) T \nabla \cdot \mathbf{v}$$

- *divergence of* $\mathbf{B}$

$$\nabla \cdot \mathbf{B} = 0$$

linear MHD stability codes:

- `CASTOR_FLOW`: compressible, resistive, toroidal rotation, full set of equations, normal mode analysis (Huysmans 1991, Kerner el al. 1998, Strumberger 2005)
- `MISHKA`: incompressible, ideal, diamagnetic effects, reduced set of equations, normal mode analysis (Huysmans et al., Phys. Plasmas 8; 2001)
- `ELITE`: compressible, ideal, toroidal rotation, solves Euler equations derived from perturbed energy δW-ansatz (Wilson et al., Phys. Plasmas; 2002)

Toroidal Rotation

Aim: Study the influence of sheared toroidal rotation of the form

$$\mathbf{V_0} = R^2\Omega_0(s)\,\nabla\phi$$ **, i.e. contravariant component** $V_0^3 = \Omega_0(s)$

on the linear stability of the plasma edge for subsonic toroidal rotation velocities.

available tools:

- `CASTOR_FLOW`: toroidal and poloidal rotation; for low to intermediate toroidal mode numbers ($n \leq 25$) or simple equilibria (E. Strumberger)
- `ELITE`: toroidal rotation; for intermediate to high toroidal mode numbers ($n \geq 8$) (P.B. Snyder)
- `MISHKA_FLOW`: toroidal rotation; for low to intermediate toroidal mode numbers (I. Chapman,S. Saarelma)

Linear MHD Stability Codes

Linearization of MHD equations with perturbation ansatz

plasma variables $F = F_0 + \tilde{F}$

equilibrium part F_0

perturbed part $\tilde{F} = \sum c_{n,m} \exp(im\theta) \times \exp(in\phi) \exp(\lambda t)$

complex eigenvalue $\lambda = \gamma + i\omega$
with growth rate γ and mode frequency ω

no linear coupling of toroidal modes n because of axisymmetry

CASTOR_FLOW

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- normal mode analysis
- solves large matrix equation iteratively
- full MHD system with 8 plasma variables ($\tilde{\rho}$, $\tilde{T}$, $\tilde{\mathbf{v}}$, $\tilde{\mathbf{B}}$), or reduced MHD system with 7 plasma variables ($\tilde{p}$, $\tilde{\mathbf{v}}$, $\tilde{\mathbf{B}}$)
- toroidal mode numbers on 4GB machine:
  $n \lesssim 10$ for experimental equilibria and high edge q,
  $n \lesssim 60$ for moderate q analytical equilibria

full MHD system: (Strumberger et al. NF **45**, 2005)

- *continuity equation*

$$\lambda\tilde{\rho} = -\rho_0 \nabla \cdot \tilde{\mathbf{v}} - \tilde{\mathbf{v}} \cdot \nabla \rho_0 - \mathbf{v_0} \cdot \nabla \tilde{\rho}$$

- *temperature equation*

$$\lambda \tilde{T} = -\tilde{\mathbf{v}} \cdot \nabla T_0 - \mathbf{v_0} \cdot \nabla \tilde{T} - (\Gamma - 1)\, T_0 \nabla \cdot \tilde{\mathbf{v}}$$

CASTOR_FLOW (continued)

- *momentum equation*

$$\lambda\rho_0\tilde{\mathbf{v}} = -\tilde{\rho}(\mathbf{v_0}\cdot\nabla)\,\mathbf{v_0} - \rho_0(\tilde{\mathbf{v}}\cdot\nabla)\,\mathbf{v_0} - \rho_0(\mathbf{v_0}\cdot\nabla)\,\tilde{\mathbf{v}} - \nabla\left(\rho_0\tilde{T}\right)$$

$$-\nabla(\tilde{\rho}T_0) - \nabla\cdot\tilde{\Pi} + (\nabla\times\mathbf{B_0})\times\tilde{\mathbf{B}} + \left(\nabla\times\tilde{\mathbf{B}}\right)\times\mathbf{B_0}$$

- *Ohm's law*

$$\lambda\tilde{\mathbf{B}} = \nabla\times\left(\mathbf{v_0}\times\tilde{\mathbf{B}} + \tilde{\mathbf{v}}\times\mathbf{B_0} - \eta_0\nabla\times\tilde{\mathbf{B}}\right)$$

with perturbed viscous force for ion Landau damping

$$-\left(\nabla\cdot\tilde{\Pi}\right)_m = -\kappa_{\|}|k_{\|}v_{\rm thi}|\rho_0\left(v_{\|}\right)_m$$

where $k_{\|} = (n - m/q)\,/R$ is the wave vector and $\kappa_{\|} \approx \sqrt{\pi}$

reduced MHD system:

neglect perturbed centrifugal force $-\tilde{\rho}(\mathbf{v_0} \cdot \nabla)\,\mathbf{v_0}$ in momentum equation
replace continuity and temperature equation by

- *pressure equation*

$$\lambda \tilde{p} = -\tilde{\mathbf{v}} \cdot \nabla p_0 - \mathbf{v_0} \cdot \nabla \tilde{p} - \Gamma p_0 \nabla \cdot \tilde{v}$$

and

- *reduced momentum equation*

$$\begin{aligned}\lambda \rho_0 \tilde{\mathbf{v}} &= -\rho_0(\tilde{\mathbf{v}} \cdot \nabla)\,\mathbf{v_0} - \rho_0(\mathbf{v_0} \cdot \nabla)\,\tilde{\mathbf{v}} - \nabla \cdot \tilde{\Pi} \\ &\quad +(\nabla \times \mathbf{B_0}) \times \tilde{\mathbf{B}} + \left(\nabla \times \tilde{\mathbf{B}}\right) \times \mathbf{B_0}\end{aligned}$$

memory requirements and CPU time scale with square of number of variables

- normal mode analysis
- solves Euler equations derived from δW-ansatz by shooting method
- force balance equation for the plasma displacement ξ with expansion in $1/n$, keeping terms up to second order
- intermediate to high toroidal mode numbers $n \gtrsim 8$
- uses poloidal harmonic localization for efficiency

MHD system in terms of ξ:

$$
\begin{aligned}
\Gamma \tilde{\rho} &= -\gamma \left[(\xi \cdot \nabla)\, \rho_0 + \rho_0 (\nabla \cdot \xi) \right] \\
\Gamma \tilde{p} &= -\gamma \left[(\xi \cdot \nabla)\, p_0 + g p_0 (\nabla \cdot \xi) \right] \\
\Gamma \tilde{\mathbf{B}} &= \gamma \nabla \times (\xi \times \mathbf{B_0}) + \frac{\gamma R^2 \Omega'}{\Gamma} \left[[\nabla \times (\xi \times \mathbf{B_0})] \cdot \nabla \psi \right] \nabla \phi
\end{aligned}
$$

with Doppler shifted growth rate $\Gamma = \gamma + in\Omega$, ratio of specific heats g and $\Omega' = \mathrm{d}\Omega/\mathrm{d}\psi$

MISHKA_FLOW

- normal mode analysis
- based on reduced linear MHD eigenmode code MISHKA_D
- reduced MHD system with 7 plasma variables ($\tilde{p}$, $\tilde{\mathbf{v}}$, $\tilde{\mathbf{B}}$)
- intermediate toroidal mode numbers $n \lesssim 40$ for moderate to high q values
- includes diamagnetic drift terms (Huysmans et al. PoP **8**(10), 2001)
- code recently developed by Ian Chapman (UKAEA Culham)

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Benchmarks

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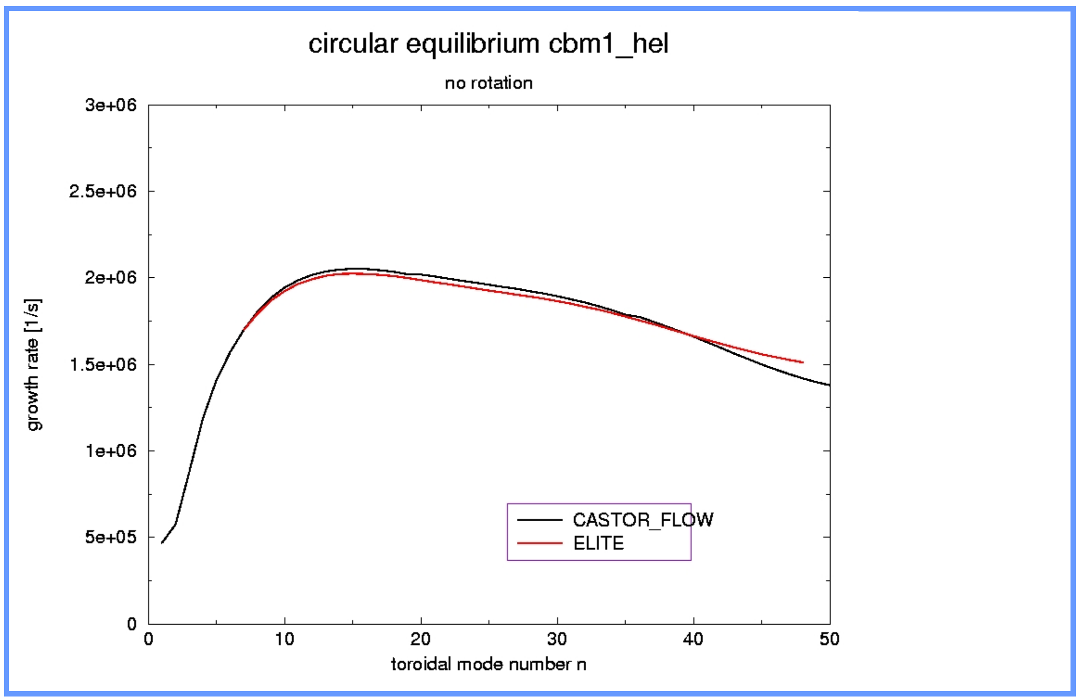
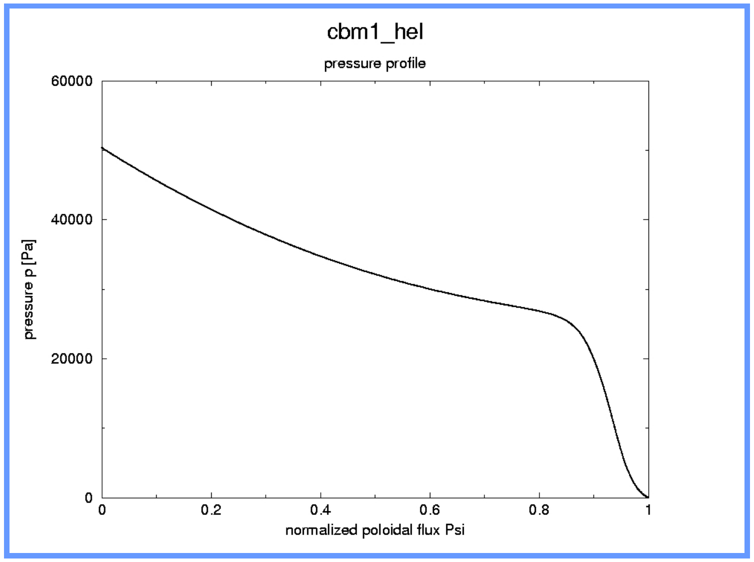
Aims:

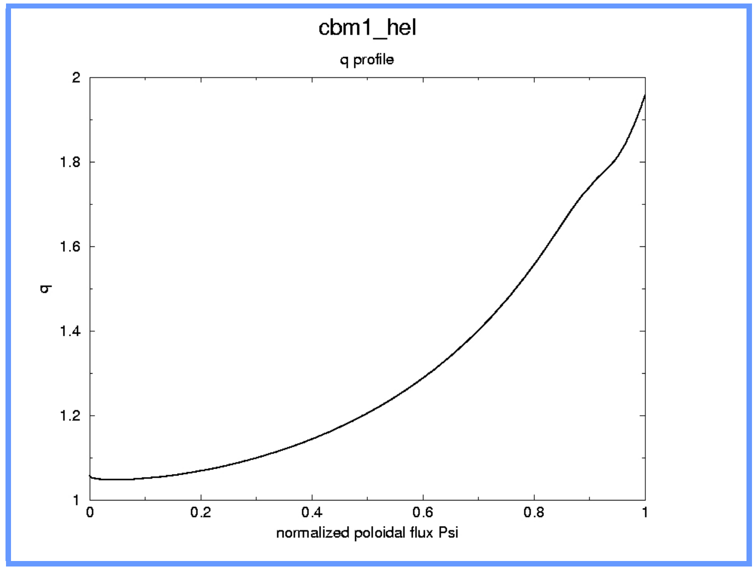
- study influence of sheared toroidal rotation on stability limits
- use CASTOR_FLOW for low to intermediate mode numbers and ELITE for high mode numbers
- compare stabilization by sheared toroidal rotation to stabilization by diamagnetic drifts (MISHKA_D)

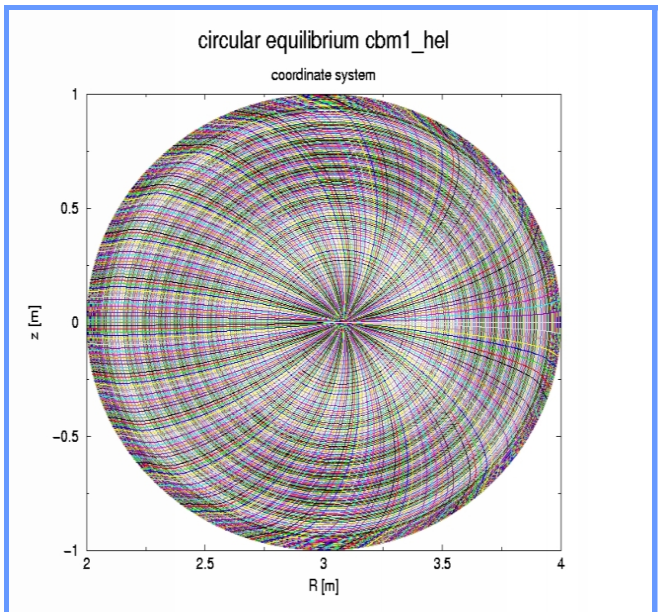


benchmark CASTOR_FLOW vs. ELITE for an ideal plasma case (no rotation)

Circular Equilibrium cbm1_hel



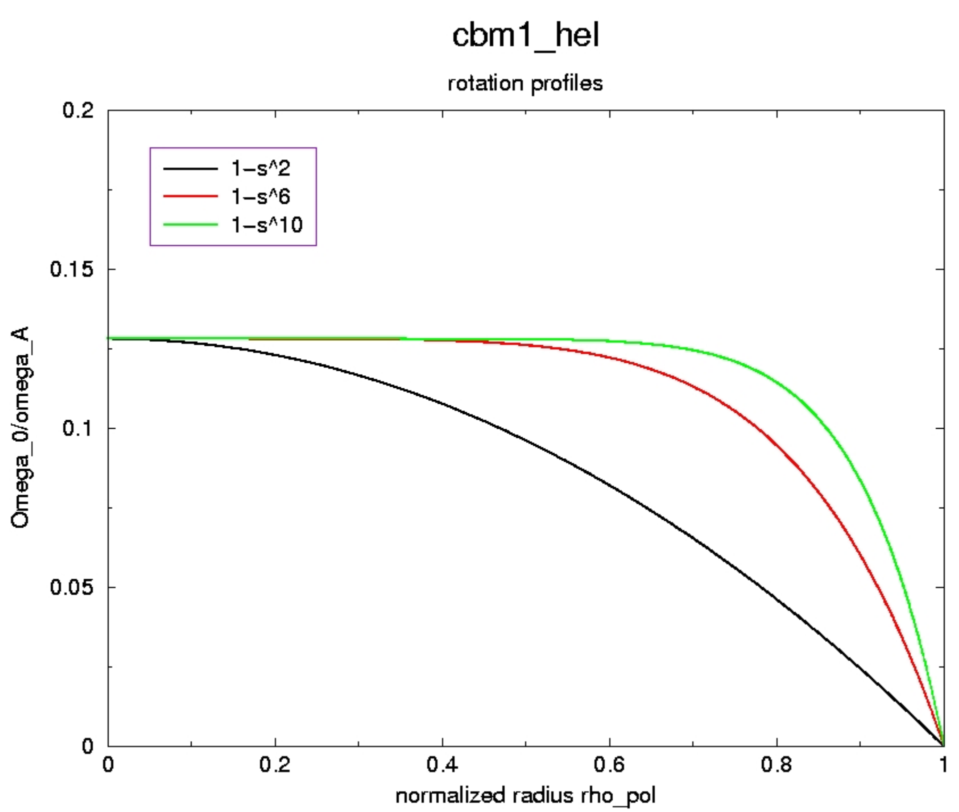




cbm1_hel:

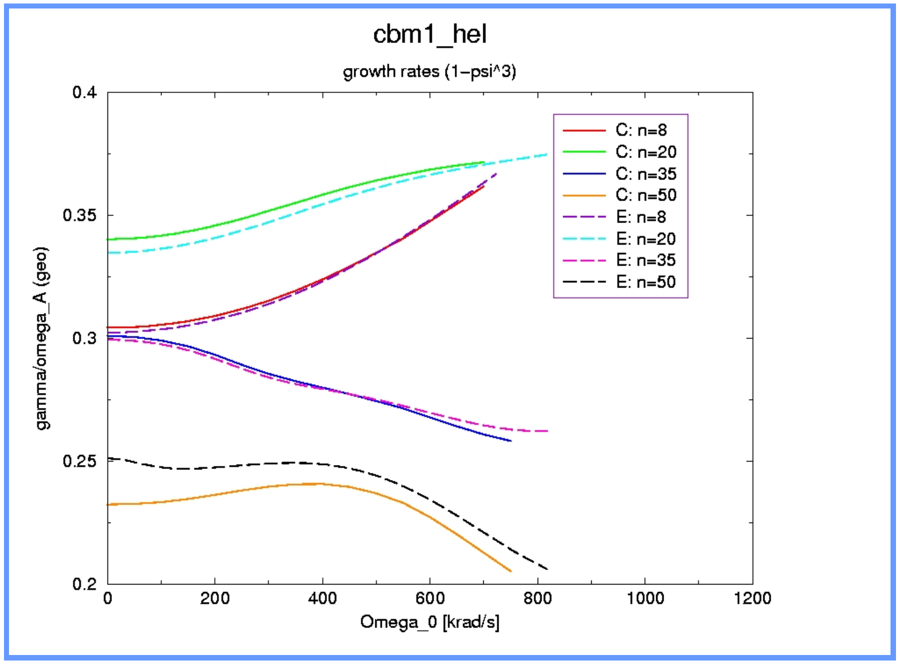
- ballooning unstable circular equilibrium
- well resolved
- low edge-q ≈ 2
- no shaping
- => weak poloidal coupling
- benchmark possible up to high toroidal mode numbers $n \sim 40$

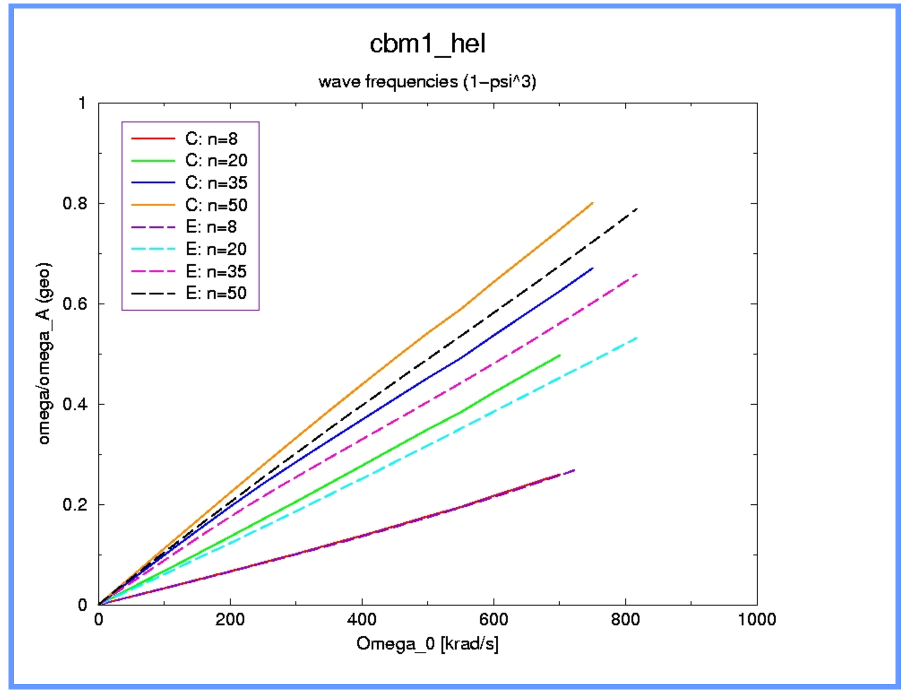
Shear Profiles



rotation profile $\Omega(s) = \Omega_0(1 - s^\chi)$
with $s = \sqrt{\psi/\psi_{\rm bd}}$ and $\chi = 2,\ 6,\ 10$

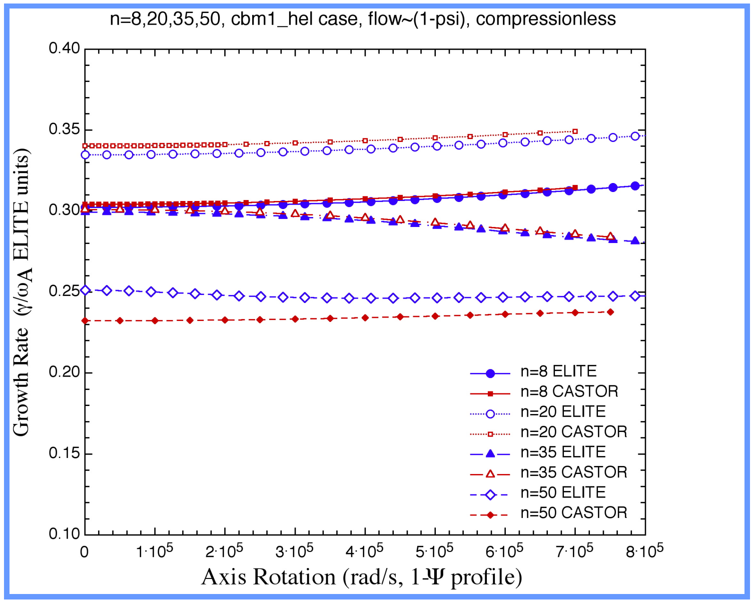
Benchmark: cbm1_hel

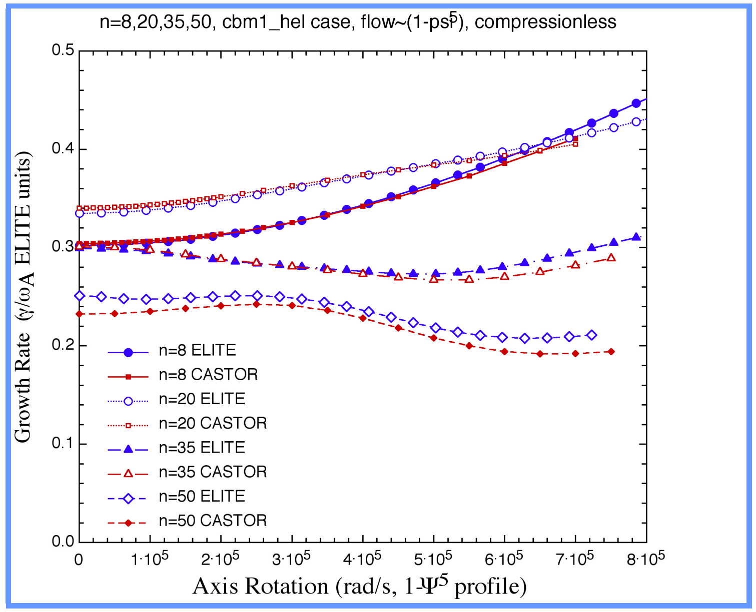


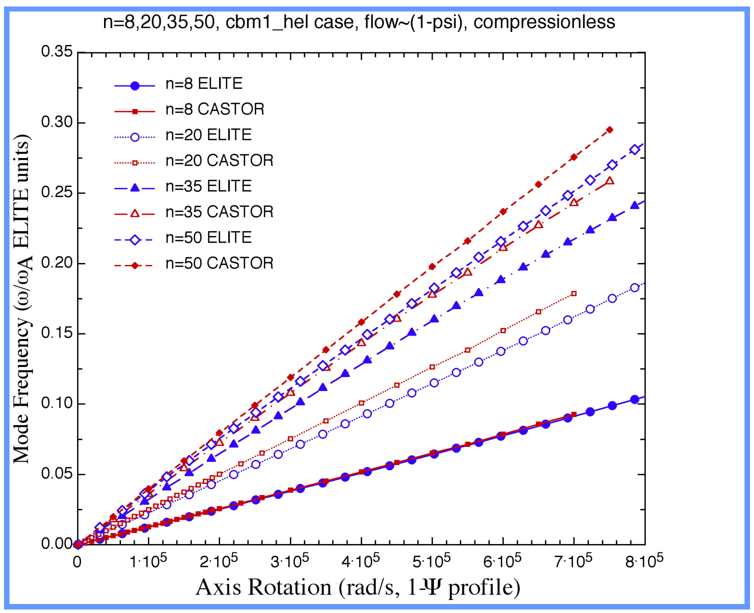


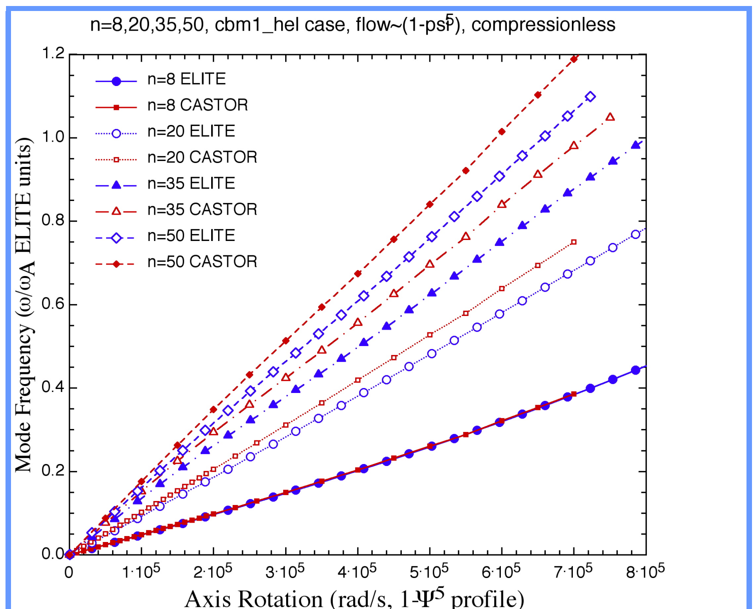
- excellent agreement of `CASTOR_FLOW` and `ELITE` for growth rates
- except for $n = 50$ (only 31 poloidal modes in `CASTOR_FLOW`)
- mode frequencies agree very well for $n = 8$
- but constant factors for $n = 20, 35, 50$ (normalization issues?)

Benchmark: cbm1_hel (continued)

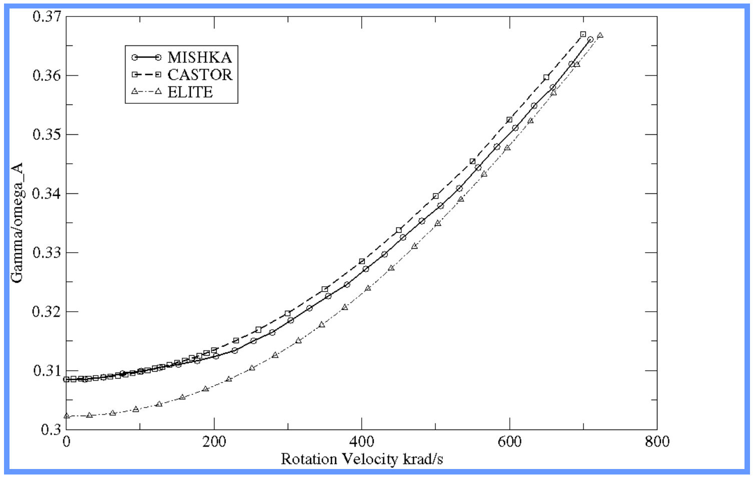


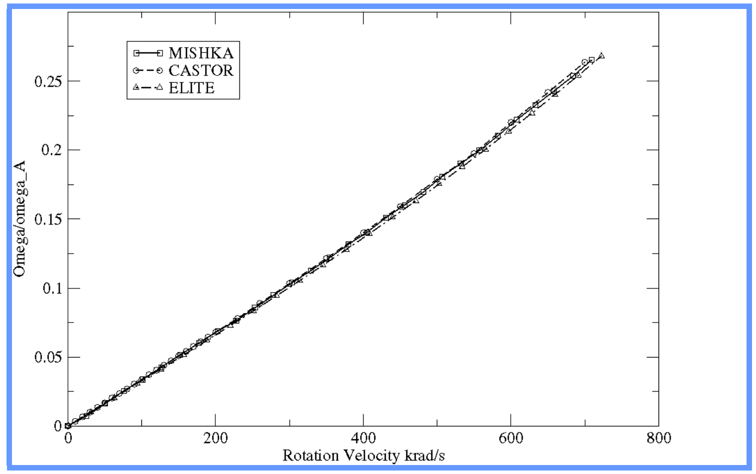


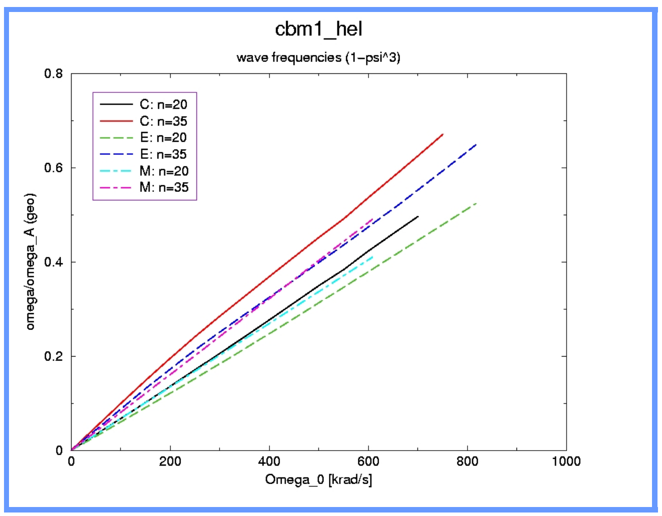




Benchmark cbm1_hel (continued)



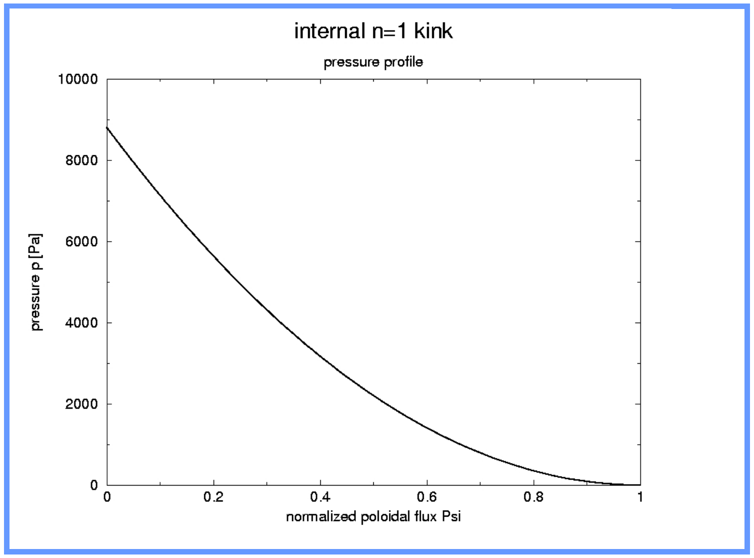


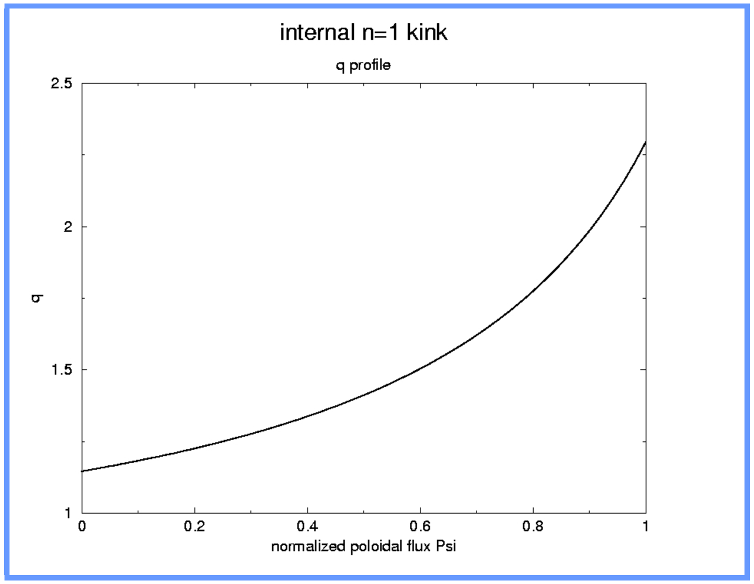


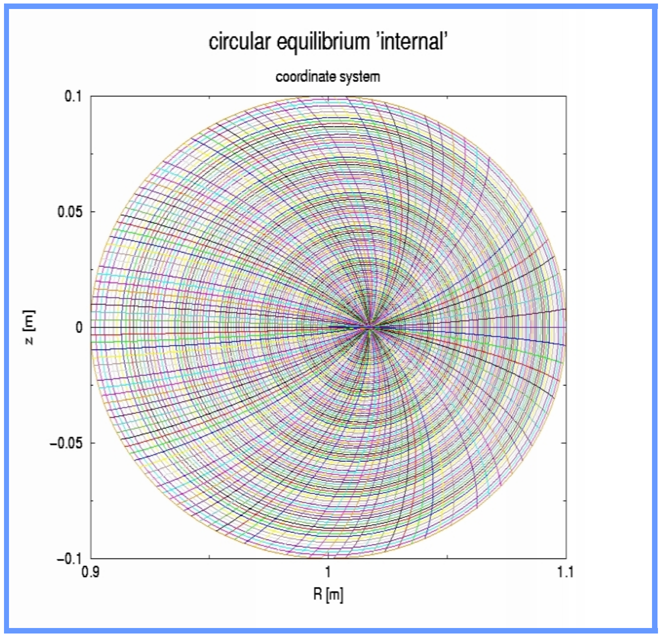
MISHKA_FLOW:

- excellent agreement for $n = 8$
- good agreement for $n = 20$ with CASTOR_FLOW mode frequencies
- good agreement for $n = 35$ with ELITE mode frequencies
- possible problem with CASTOR_FLOW wave frequencies for high n but very good agreement for growth rates

Circular Equilibrium 'internal kink'







internal $n = 1$ kink:

- circular Bussac equilibrium
- unstable to $n = 1$ internal kink
- fixed boundary (no vacuum)
- no shaping, low edge-q ≈ 2.5
- $=>$ weak poloidal coupling

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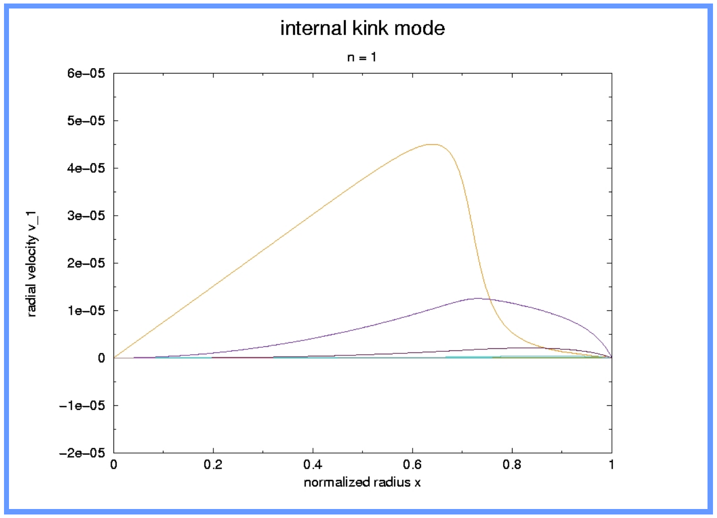
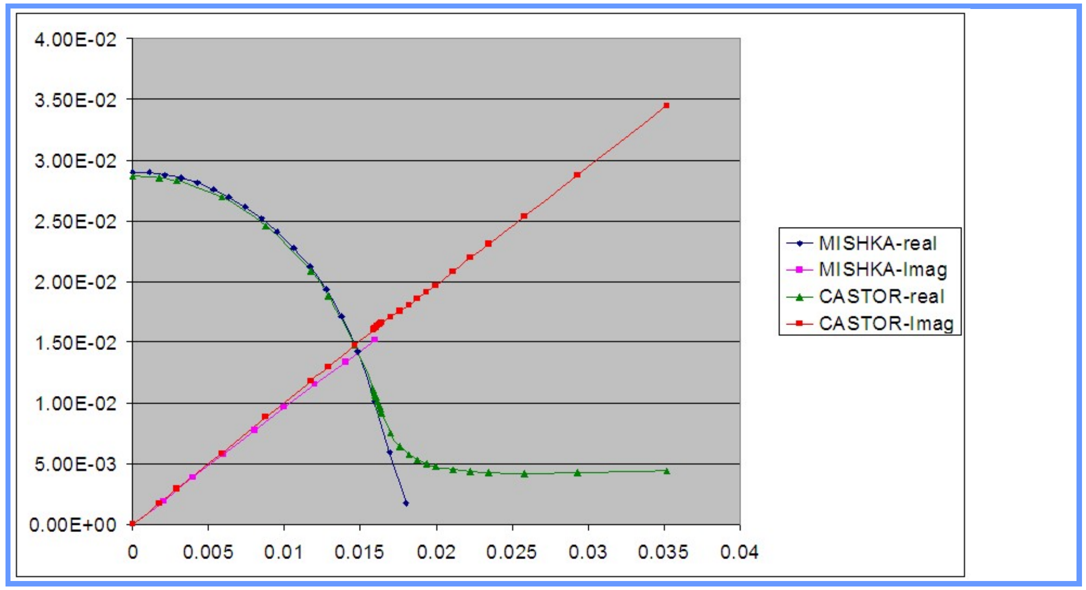
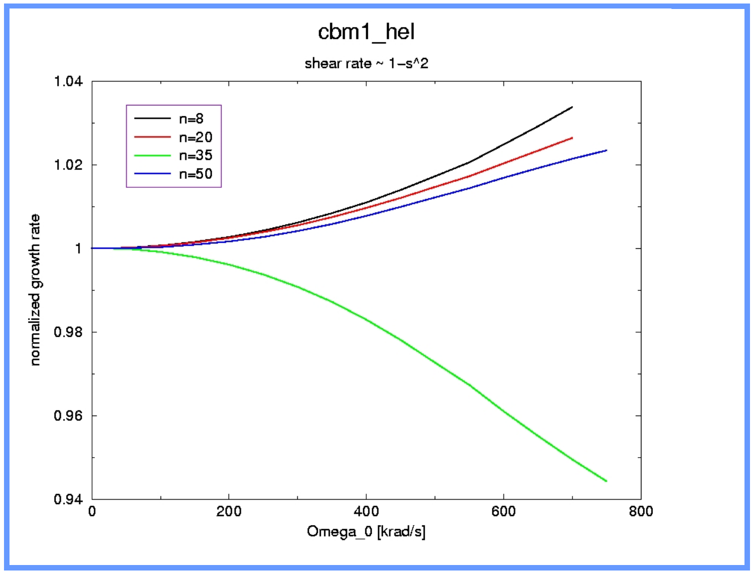
Benchmark: 'internal kink'

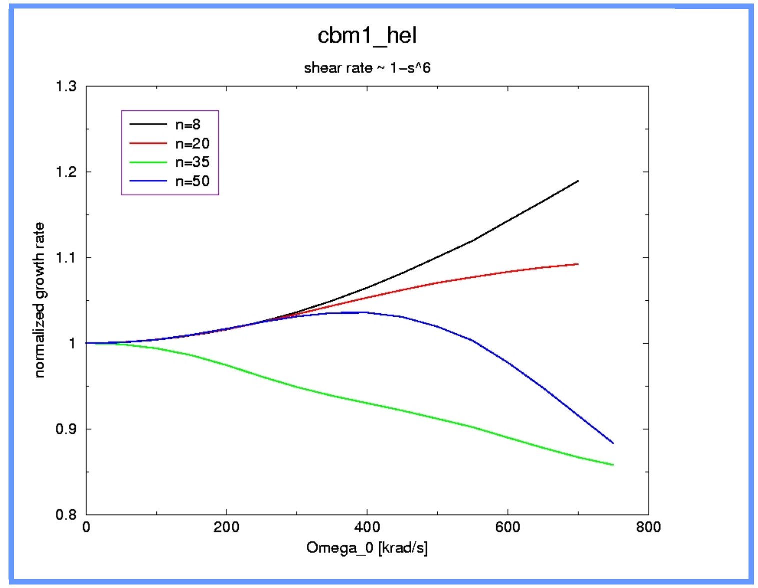


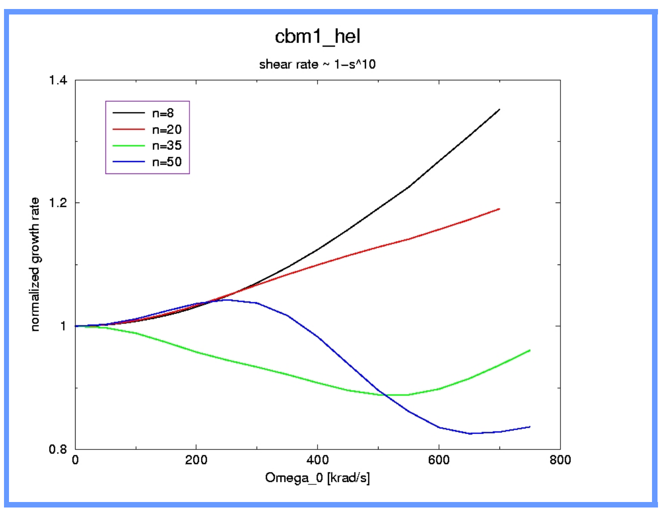


- rigid rotation (no shear)
- excellent agreement in growth rates and mode frequencies between MISHKA_FLOW and CASTOR_FLOW
- strong stabilization of the internal $n = 1$ kink
- completely stabilized in MISHKA_FLOW, but finite growth rate in CASTOR_FLOW
- possibly due to missing viscous damping (to be tested), sound waves?, or destabilizing effect of centrifugal force

Toroidal Rotation: First Results



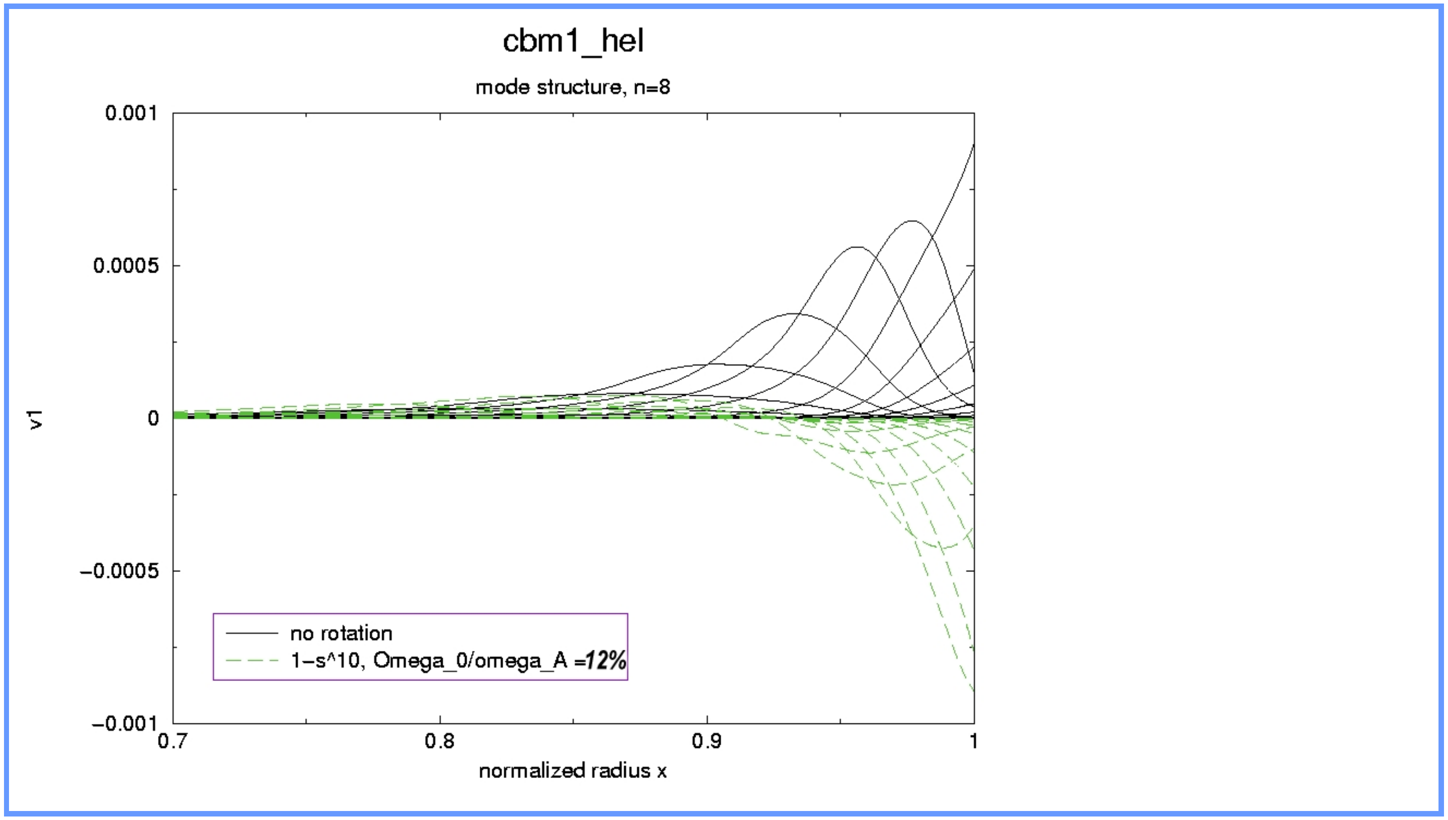




Results:

- stronger effects on high-n modes
- low shear generally destabilizing (centrifugal force)
- intermediate shear generally stabilizing
- high shear again destabilizing (Kelvin-Helmholtz?)
- localization of shear and affected mode region also matter

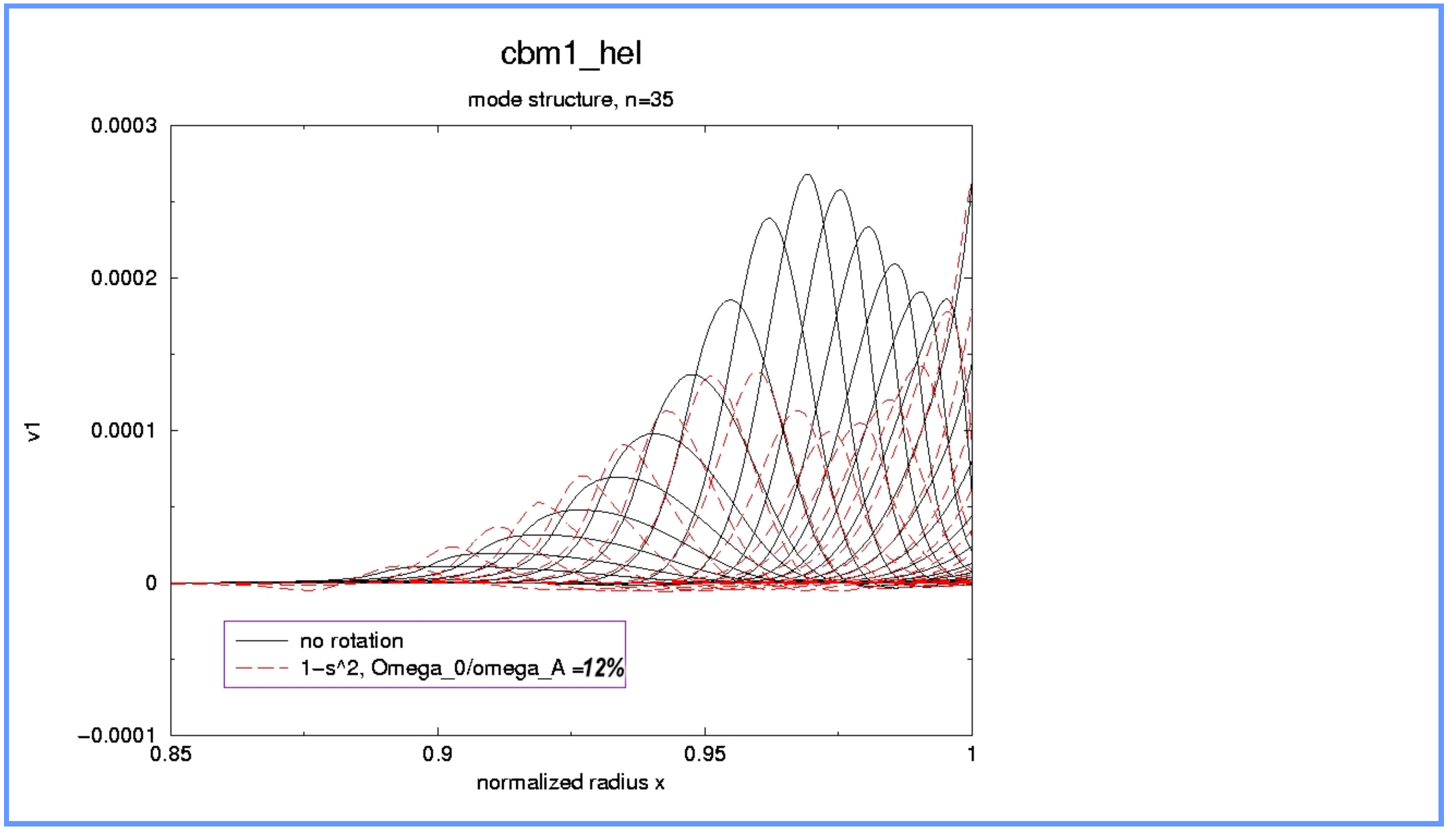
Mode Structures



Changes in Mode Structure:

- strongly destabilized mode $n = 8$ at very high shear and high rotation frequency
- no major change in mode structure
- internal modes slightly weakened
- entire mode shifted radially outward (centrifugal force)
- wide mode structure less affected by shear

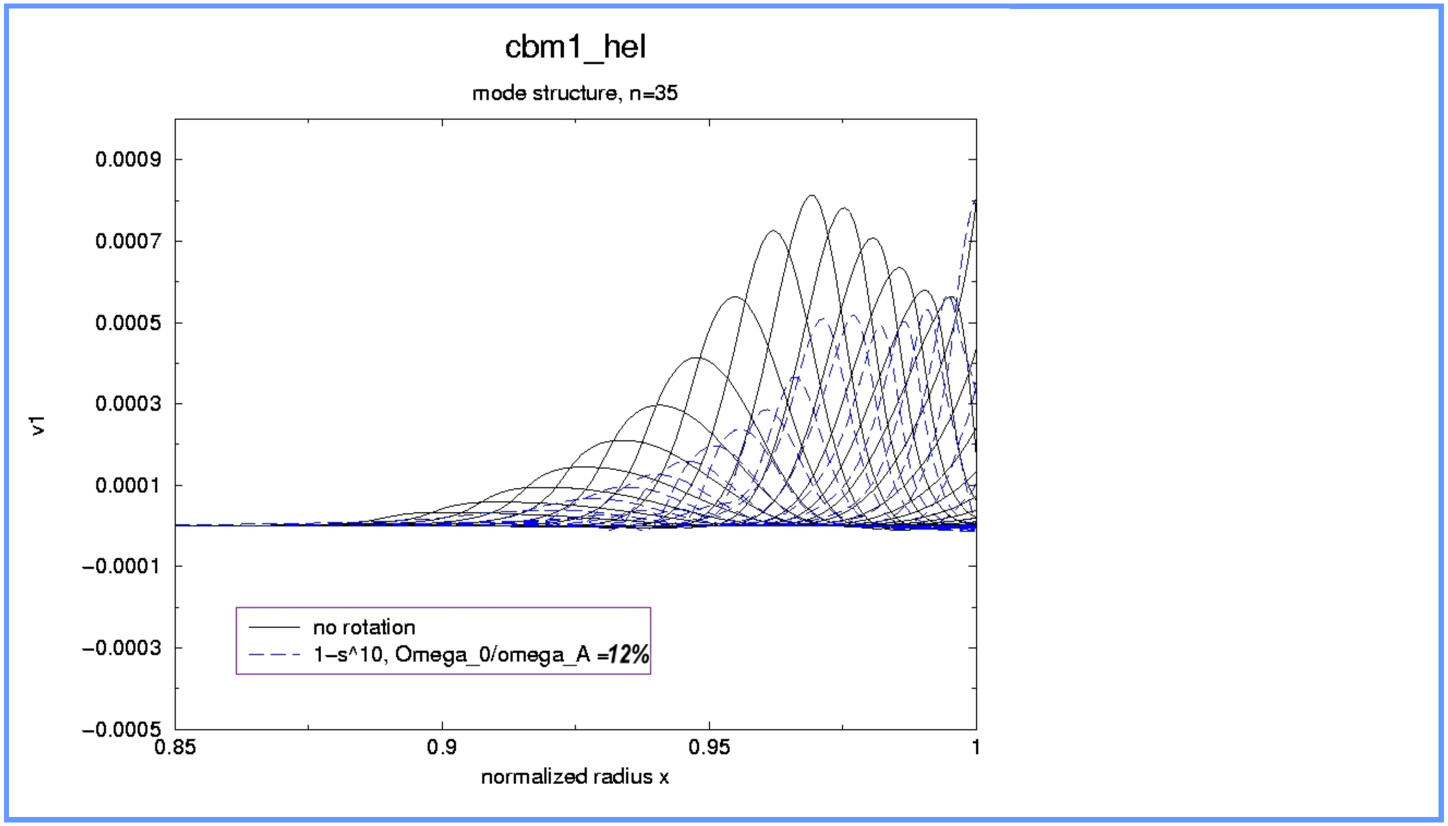
Mode Structures (continued)



Changes in Mode Structure:

- weakly stabilized mode $n = 35$ at intermediate shear and high rotation frequency
- mode structure is smeared out and broadened
- internal modes strongly reduced
- mode center shifted radially inward (shear larger at edge)
- narrow edge mode strongly affected by shear

Mode Structures (continued)



Changes in Mode Structure:

- nearly unaffected mode $n = 35$ at very high shear and high rotation frequency
- mode structure broadened
- internal modes strongly reduced
- mode center shifted radially outward (centrifugal force reaches farther out)
- narrow edge mode strongly affected by shear
- compensation of destabilizing and stabilizing effects

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Stabilization through Toroidal Rotation

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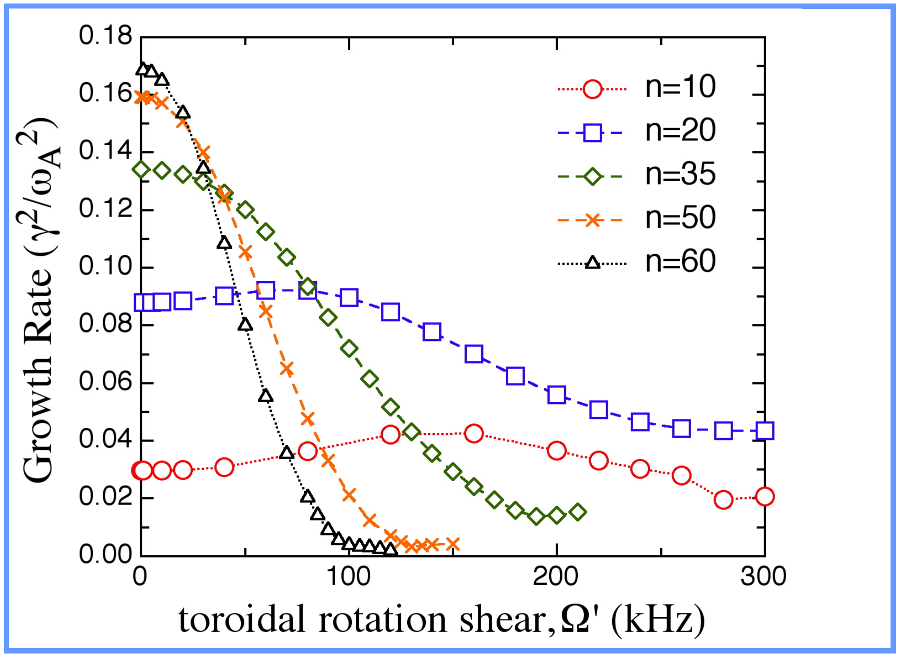
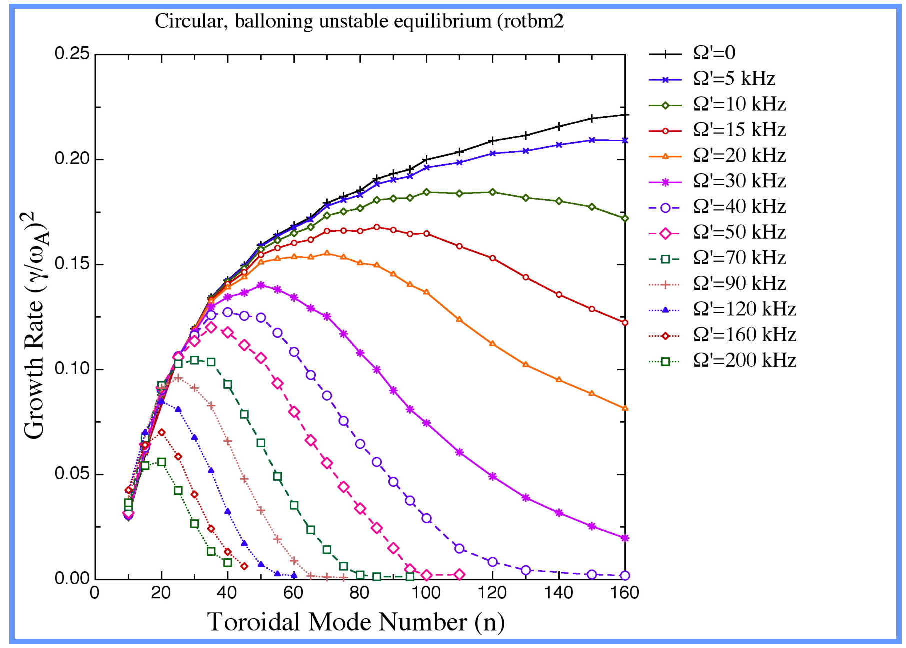


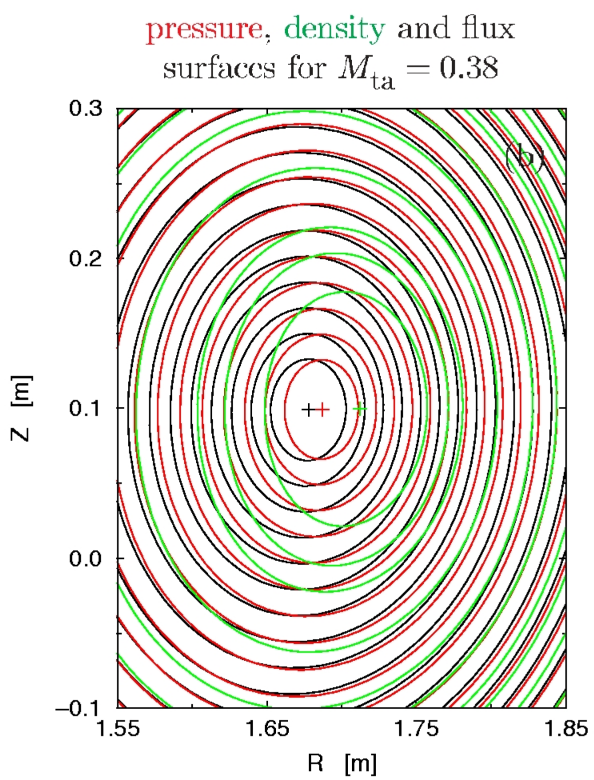


stabilization of ballooning modes through rotational shear for a circular equilibrium (P. Snyder, 2005)

Toroidal Rotation with ELITE (P. Snyder):

- circular equilibrium, ballooning unstable
- stabilization for high toroidal mode numbers or high rotational shear
- destabilizing for low toroidal mode numbers and low shear

High Rotational Mach Numbers



Strumberger et al. NF **45**, 2005

from force balance equation follows:

pressure $p(\psi, \theta) = \bar{p}(\psi) \exp\left(\frac{\Omega^2 R^2}{2c_s^2}\right)$
with sound speed $c_s = \sqrt{k_B T / m_i}$

=> for high Mach numbers $M = \Omega R / c_s \gtrsim 0.3$: pressure and density are not flux surface quantities anymore!

equilibrium code `DIVA` allows to calculate two-dimensional equilibria

`CASTOR_FLOW` allows stability analysis for equilibria with poloidally varying pressure and density

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Summary and Outlook

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Results:

- centrifugal force can be destabilizing for high rotation frequencies
- intermediate shear acts stabilizing on narrow modes
- strong shear can act destabilizing (Kelvin-Helmholtz)
- mode localization with respect to shear layer and mode extent important
- more detailed studies necessary to investigate effects of toroidal rotation

Linear Edge Stability and Toroidal Rotation:

- successful benchmark of `CASTOR_FLOW`, `ELITE`, and `MISHKA_FLOW` for circular equilibria with internal and edge localized modes
- next step: experimental equilibria (high-q, shaped)
- low-n mode studies with `CASTOR_FLOW` plus high-n mode studies with `ELITE`
- comparison of diagmagnetic drifts to toroidal rotation
- access to second stability regime by rotation effects?