

# Effect of equilibrium reconstruction on pedestal analysis (work in progress)

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+ITM/IMP-1 Equilibrium Reconstruction group:

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# Topics

- Introduction to equilibrium reconstruction
- How to do equilibrium reconstruction?
- How to improve it
- Some examples
- Can we do better?
- Future plans

# Tokamak plasma equilibrium

axisymmetric force balance:

$$\partial/\partial t=0, \partial/\partial \zeta=0$$

$$\text{Force balance: } \vec{j} \times \vec{B} = \nabla p$$

$$\text{Ampere's Law: } \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{j} = 0$$

# Representation of variables and functions

$$\begin{aligned}\vec{B} &= F\nabla\zeta + \nabla\zeta \times \nabla\Psi \\ &= \vec{B}_{\text{toroidal}} + \vec{B}_{\text{poloidal}}\end{aligned}$$

$$\Psi = (\text{Poloidal Flux})/2\pi$$

$$F = RB_{\text{toroidal}}$$

$p$  and  $F$  are flux functions,  $\bar{\Psi}$  is normalised flux

$$p = p(\bar{\Psi}), F = F(\bar{\Psi}), \quad \bar{\Psi} = (\Psi - \Psi_{\text{axis}})/(\Psi_{\text{LCFS}} - \Psi_{\text{axis}})$$

$p'$  and  $FF'$  are modelled as polynomial functions (or splines) of the normalised poloidal flux,  $\bar{\Psi}$

# Representation of variables and functions

The toroidal plasma current is represented as

$$j_{toroidal} = R p' + \frac{1}{R} \frac{FF'}{\mu_0}$$

Notice  $R$  and  $1/R$  local dependencies!

Diamagnetism:  $FF'$  has opposite sign to  $p'$

Locally,  $j_{toroidal} < 0$  if gradients are steep

The poloidal plasma current is

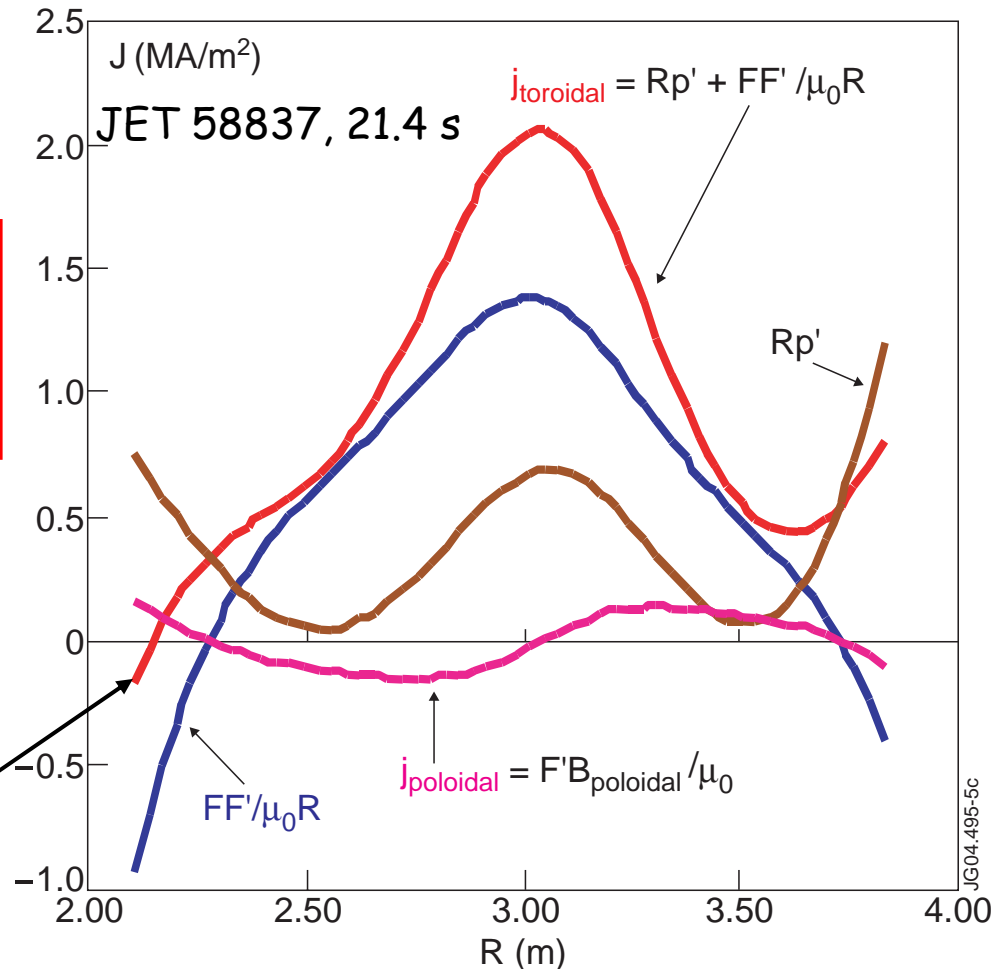
$$j_{poloidal} = -F' B_{poloidal} / \mu_0$$

# Current density profiles just before an ELM

## Edge diamagnetism

$$j_{toroidal} = R p' + \frac{1}{R} \frac{FF'}{\mu_0}$$

due to  $1/R$ ,  $j_{toroidal}$  can be negative inboard



Reconstruction of equilibrium just before ELM (tricky!!!)

# Introduction to equilibrium reconstruction

Find an equilibrium that minimizes  $\chi^2$

$$\chi^2 = \underbrace{\sum \text{fwt} \frac{[\Psi_{\text{meas}}(\mathbf{k}) - \Psi_{\text{calc}}(\mathbf{k})]^2}{\sigma(\mathbf{k})^2}}_{\text{measurements}} + \underbrace{\sum \text{fwt} [\mathbf{C}_{\text{desired}}(\mathbf{k}) - \mathbf{C}_{\text{calc}}(\mathbf{k})]^2}_{\text{constraints}}$$

- Measurements and constraints are calculated, or approximated, with Green functions

$$\Psi = \sum_{\mathbf{k}'} \mathbf{G}(\mathbf{k}, \mathbf{k}') \mathbf{I}(\mathbf{k}') + \Psi_{\text{external}}$$

$G$ 's are like mutual inductances: the flux at  $\mathbf{k}$  produced by a current at  $\mathbf{k}'$

# Fitting and equilibrium iterations

fitting: invert  $G$  matrix to compute  $I$ 's, profiles, minimize  $\chi^2$

$$\begin{pmatrix} G \end{pmatrix} \begin{pmatrix} I \\ p' \\ FF' \end{pmatrix} = \begin{pmatrix} M \\ C \end{pmatrix}$$

equilibrium: solve Grad-Shafranov equation,

$$L(\Psi) = R p'(\bar{\Psi}) + FF'(\bar{\Psi}) / R / \mu_0$$

Typically

- fitting iterations update coil currents & profiles
- equilibrium Picard iterations compute  $j(\bar{\Psi})$ , update  $\Psi(j)$ , find boundary, recompute  $\bar{\Psi}$



## Usual pedestal analysis in JET

- external magnetic and current measurements: magnetic probes, flux loops, saddle loops, currents in coils
- How much freedom does the current profile representation have? Very little

2<sup>nd</sup> order polynomials for  $p'$  and  $FF'$ :

$$p'(\Psi) \approx a_0 + a_1 \bar{\Psi} + a_2 \bar{\Psi}^2$$

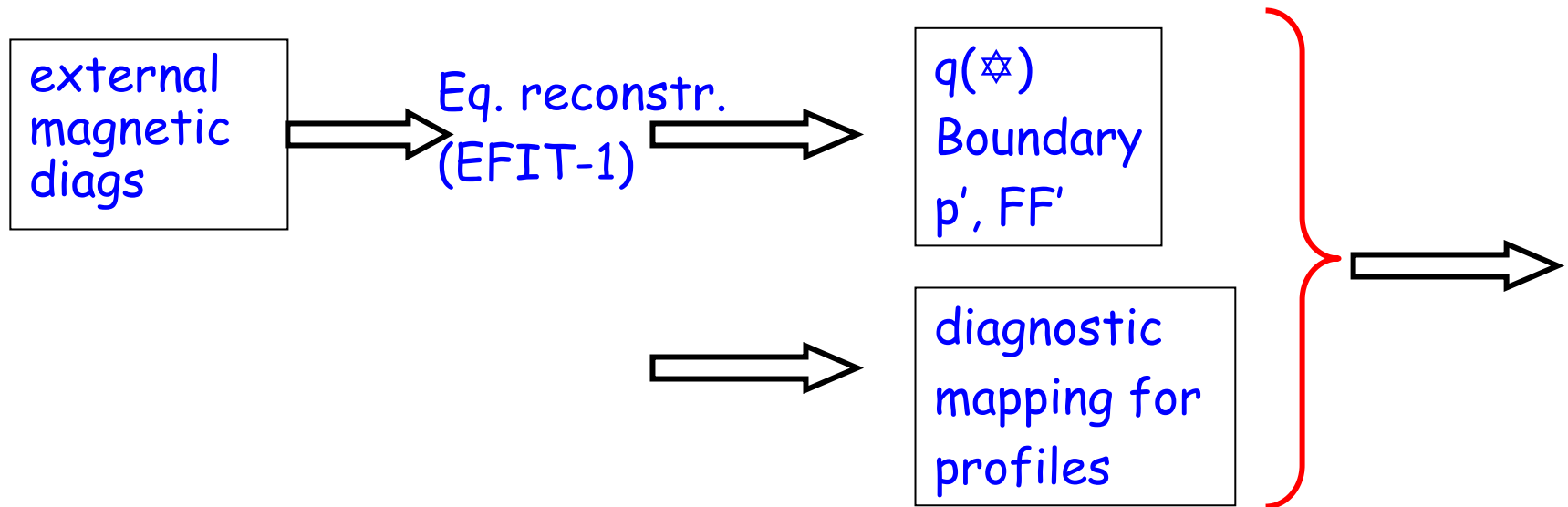
+ a regularising constraint to minimise  $p''' \sim 2a_2$

- Can this represent pressure profiles with a pedestal?

Not really: parabolic  $p(\Psi) \approx p_0 + p_1 \bar{\Psi} + p_2 \bar{\Psi}^2$

# Reconstruction of equilibrium for pedestal studies

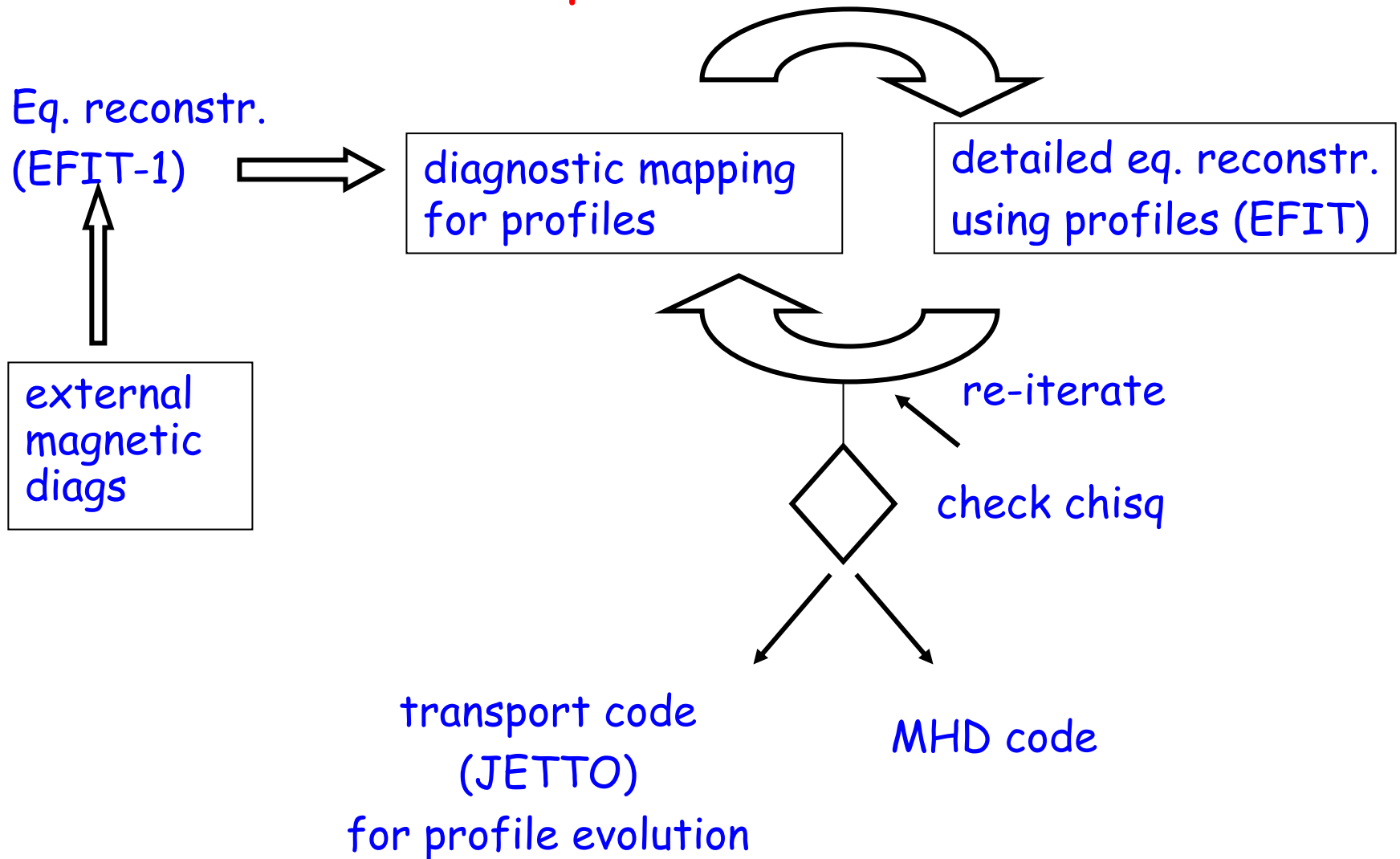
What is being done now at JET:



transport code  
(JETTO)

- Profile evolution ( $n$ ,  $T$ ,  $p$ ,  $E_r$ ).
- Refine/Update equilibrium as needed
- Refine equilibrium (HELENA)
- Test MHD (MISHKA?)

# Better: equilibrium iteration

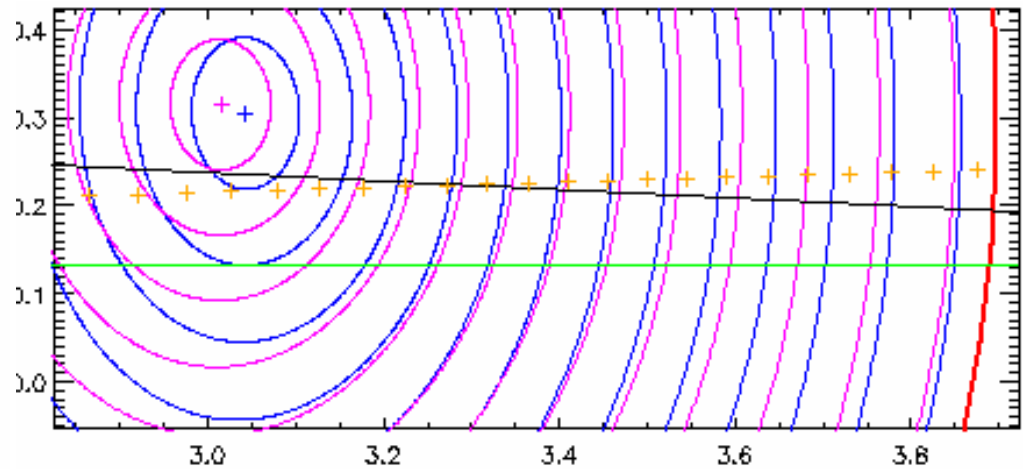
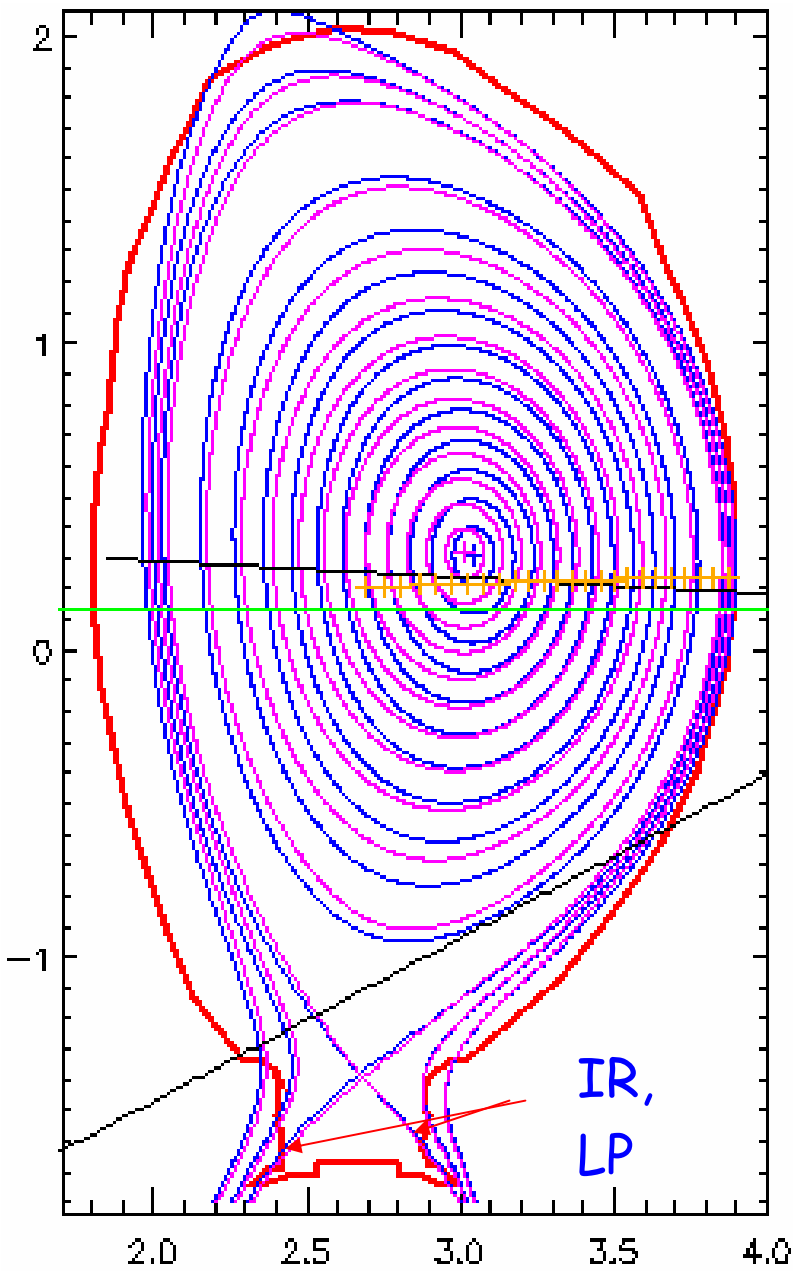


# Art of detailed equilibrium reconstruction

- Mutually incompatible measurements and constraints: many diagnostics => many answers
- Results depend on diagnostic choices, constraint fitting weights, estimated error bars
- Integral quantities ( $q$ , shear) are not as sensitive as local measurements: many very different equilibria can produce barely distinguishable  $q$  profiles
- If many measurements are available, more structure can be found in  $p$  and  $j$ .

Some useful diagnostics for equilibrium reconstruction

Electron Cyclotron Emission: ECE



Lines of sight:

Thomson Scattering:

LIDAR, edge LIDAR, HRTS

Motional Stark Effect: MSE

+ Polarimetry line integrals

## Can we do better? On the equilibrium side

EFIT can also be given more information from

- MSE: local measurements of  $B_{\text{pol}}$ , may be polluted by large electric fields
- Polarimetry: line integral of  $n_e B_{\text{pol}}$
- LIDAR + "treated"  $T_i$  info: local measurement of pressure profile
- ECE can provide information on  $B$ , if  $T_e(B)$  is known
- Core measurements really help pedestal reconstruction!

When many measurements are available



high order polynomials (or splines) can be used to represent  $p'$  and  $FF'$



higher grid resolution can be exploited



Details of  $p$  and  $j$  can be captured

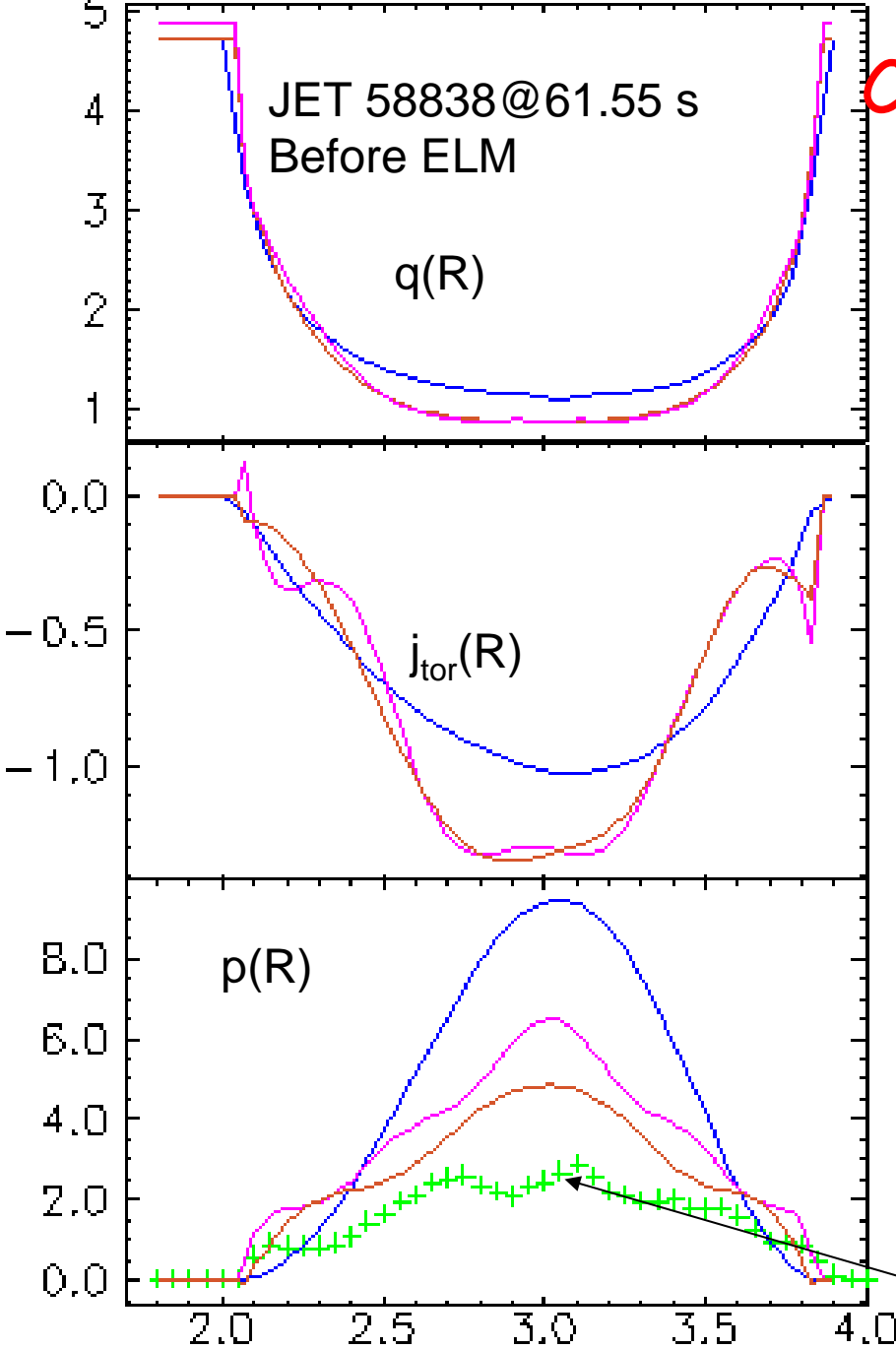
# What do we get then?

- I am still dissatisfied with the equilibrium reconstructions I have: so far it has not been possible to properly fit the various available measurements.
- Slow work: manual adjustments of data and mapping are needed, by many diagnosticians, and models should be tested, validated and optimised further (iron, coil connections)

Let's compare results for some examples so far:

1. External magnetics , 2  $p'$  and 2  $FF'$  coefs.
2. +MSE, polarimetry, 5  $p(R,Z)$  points (3.0-3.77 m, LIDAR and edge LIDAR), Ti(CX), 5  $p'$  , 5  $FF'$  coefs
3. +MSE, polarimetry, 9  $p(R,Z)$  points (2.5-3.65 m), Ti=Te, 5  $p'$  , 5  $FF'$  coefs, reduced fwt for position feedback coils, extra shift of LIDAR.

# Comparing reconstructions



1. Blue: chain1 EFIT
2. Orange: + MSE, polarimetry, pressure
3. Pink: + extra-in-shifted pressure

$q$  profile:  
not particularly different at pedestal

- Toroidal current density
1. Smooth, small at edge
  2. Considerable  $j > 0$  at edges
  3.  $j > 0$  outer edge,  $j < 0$  inner edge

- Pressure
1. smooth pressure, no pedestal
  2. pedestal, but  $p < 0$  at edge
  3. pedestal, shifted inwards

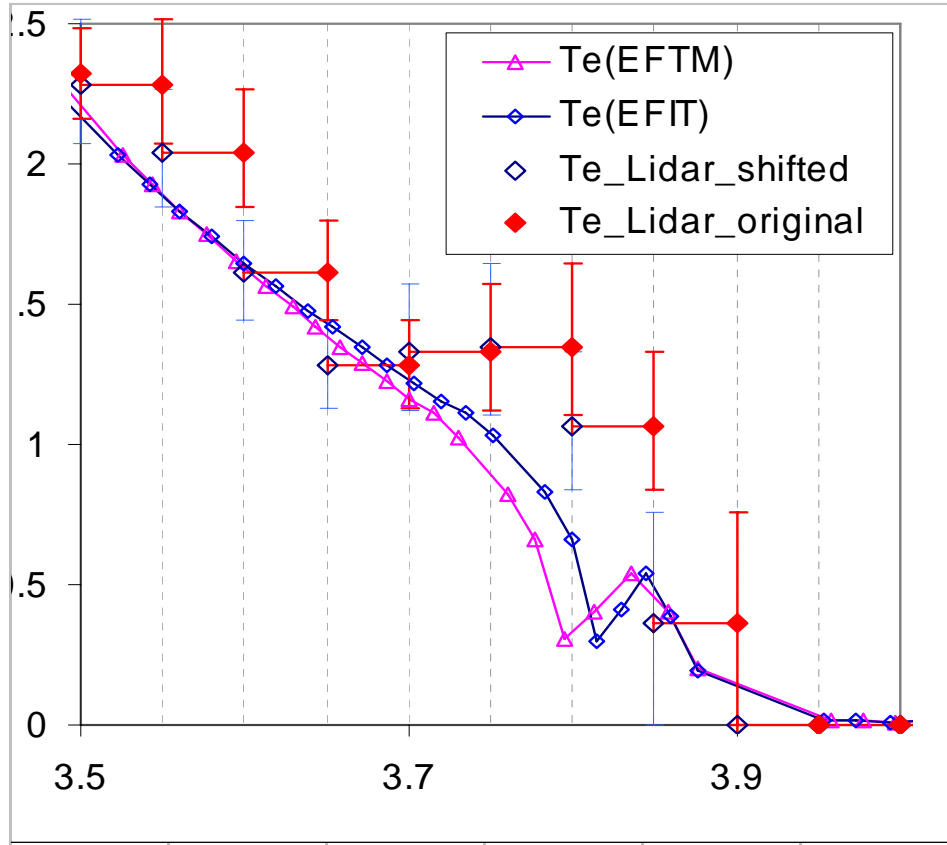
"data, from 1<sup>st</sup> mapping"



# Comparing reconstructions: pedestal

Blue: TeECE with chain1 EFIT

Pink: TeECE with reconstruction 3  
(E. de la Luna)



Red points: original LIDAR

Blue points: 5 cm shifted LIDAR

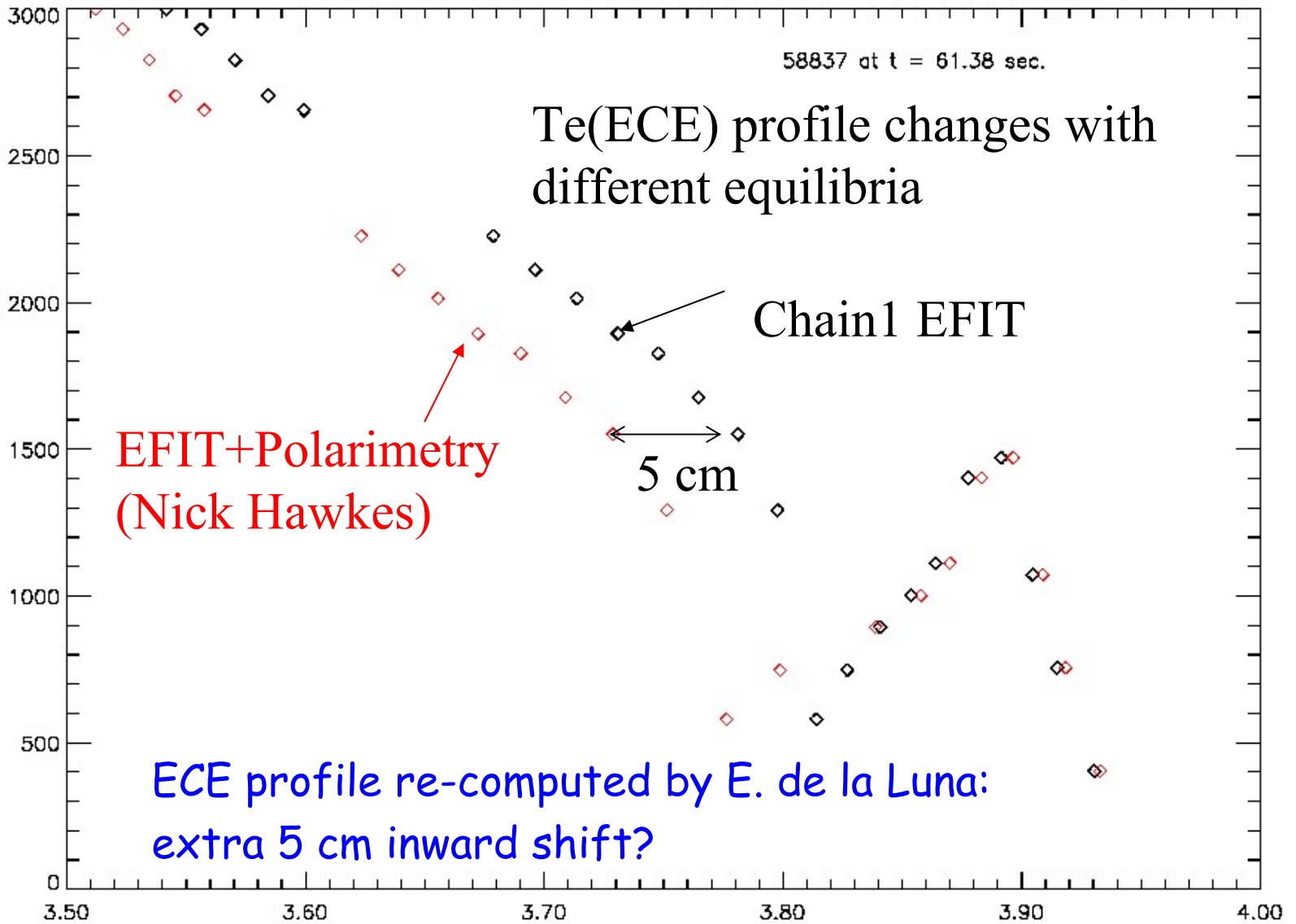
In 2000 LIDAR was shifted inward 5 cm “to match Te\_ECE”.

Te\_ECE profile depends on equilibrium reconstructed  $B_{tot}$ .

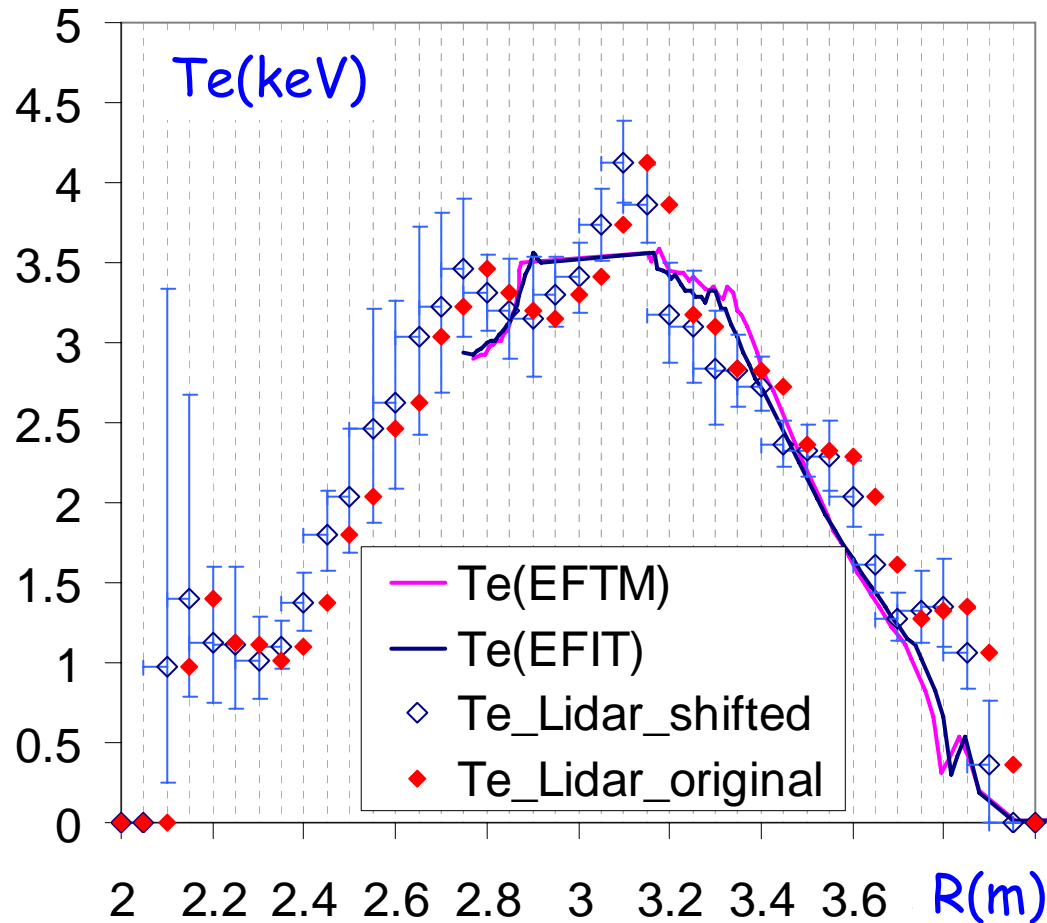
But, my best reconstruction so far requires an extra 2 cm shift?

Similar effects for reflectometry.

Also, wrong strike locations in all cases!



## Overall profile: Not just a simple shift



In the edge, an  
"extra" shift?

Core original LIDAR  
position better  
match to ECE

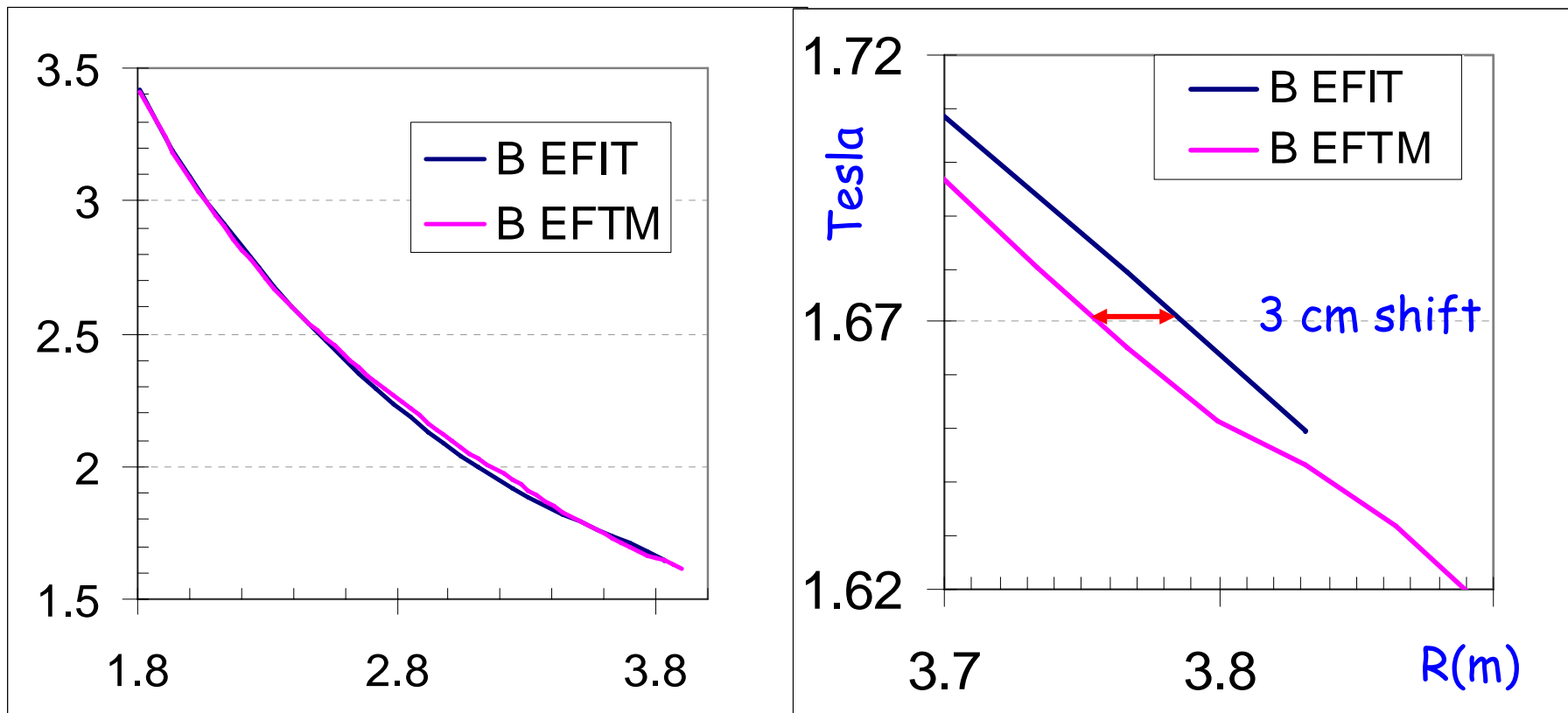
Conclusion:

I still don't trust  
the reconstructions.

Probably need to find  
another shot, w/o MHD,  
but with many diags.

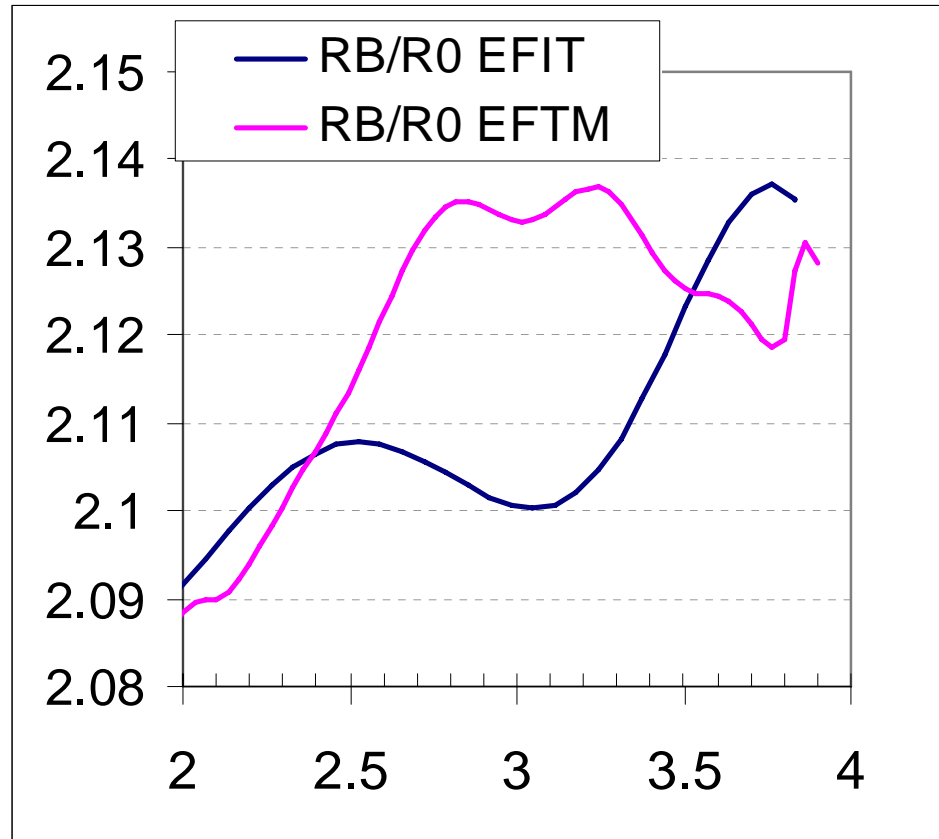
# Diagnostic mapping

value of B for ECE and reflectometry  
Here, plot B from EFIT along equator



# Diagnostic mapping

value of B for ECE and reflectometry  
Here, plot B from EFIT along equator



## More to do? In EFIT

- Refine mesh: chain1 efrit has grid mesh of 7x12 cm (65x65) or 14x24 cm (33x33): this must be improved for detailed reconstruction!  
MSE resolves 5 cm, pedestal widths are a few cm, current density profile can be even sharper.
- V&V iron core geometry model, iron permeability model (with dry runs, comparison with FE codes).
- Reorganize coil connections in code, to take into account real systems for position and shape control.
- Add diagnostics and constraints.
- Treat time and spatial averages consistently.

## More to do? In transport codes, JETTO?

Recap:  $q$ ,  $p'$  and  $FF'$  are taken from EFIT and used by JETTO, via HELENA, then evolved.

A proposal:

- read out the eventual JETTO-produced equilibrium
- compute with it  $\chi^2$  of the available data, as in EFIT.
- if  $\chi^2$  is reasonable, proceed.  
If not, iterate detailed equilibrium reconstruction with more diagnostic information, more structure in  $p'$ ,  $FF'$ . Then get back to JETTO

# Summary

Work is underway to improve detailed equilibrium reconstruction at JET

Flux-mapping of diagnostics is far from trivial

Core diagnostics can help pedestal mapping

Propose evaluating  $\chi^2$  in JETTO before proceeding to MHD analysis.

PS:

ITM IMP-1 project is equilibrium reconstruction.

If there is interest in it, I can say a few words about it.



# Equilibrium reconstruction at JET

- Up until now EFITJ: from original EFIT by Lang Lao code, changed for JET by Dennis O' Brien, Wolfgang Zwingman and Vladimir Drozdov.
- A re-write of EFIT into fortran90 (mostly) was done in Culham by Lynton Appel.
- Lynton, Vladimir and myself working on unification for chain1 use

# ITM/IMP-1 Project

Project Leader: Guido Huysmans

Deputy Project Leader: Lynton Appel

## Aims:

- define input-output interfaces for equilibrium codes
- unify EFIT
- V&V ITM equilibrium reconstruction codes

# Equilibrium reconstruction at JET

- Where are we now? *In the middle of it.*
- Data read from EFITJ/K-file, translated to different formats.
- Code now works and agrees exactly with JET for external magnetic measurements without saddles.
- Ongoing work: PPF I/O, more diagnostics, merging of various development versions, refine grid, iron model, circuit descriptions,...

# Equilibrium reconstruction: ITM and CREATE

- In parallel, Wolfgang Zwingman is developing EFIT-ITM in Matlab.
- Paddy McCarthy has various codes for ASDEX, with great capabilities (SOL currents, regularised high order polynomials,  $q$  constraints, etc)
- The CREATE group has a suite of finite element equilibrium codes, with detailed treatment of the iron core (geometry and magnetization), now improving internal magnetic diagnostics.
- ITM-Equilibrium reconstruction:  
A meeting is planned sometime this spring

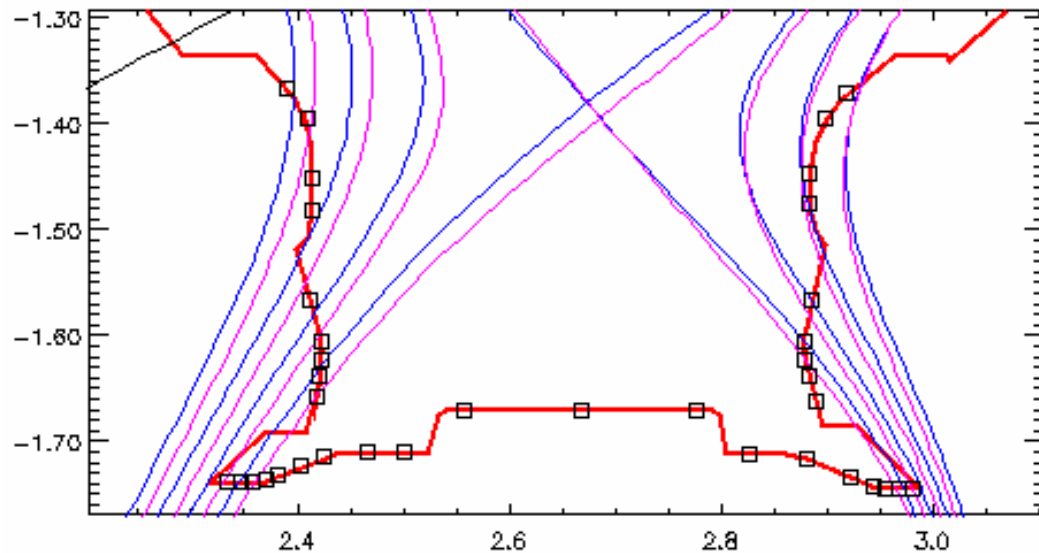
# Introduction to equilibrium reconstruction

What measurements could be used for the fits?

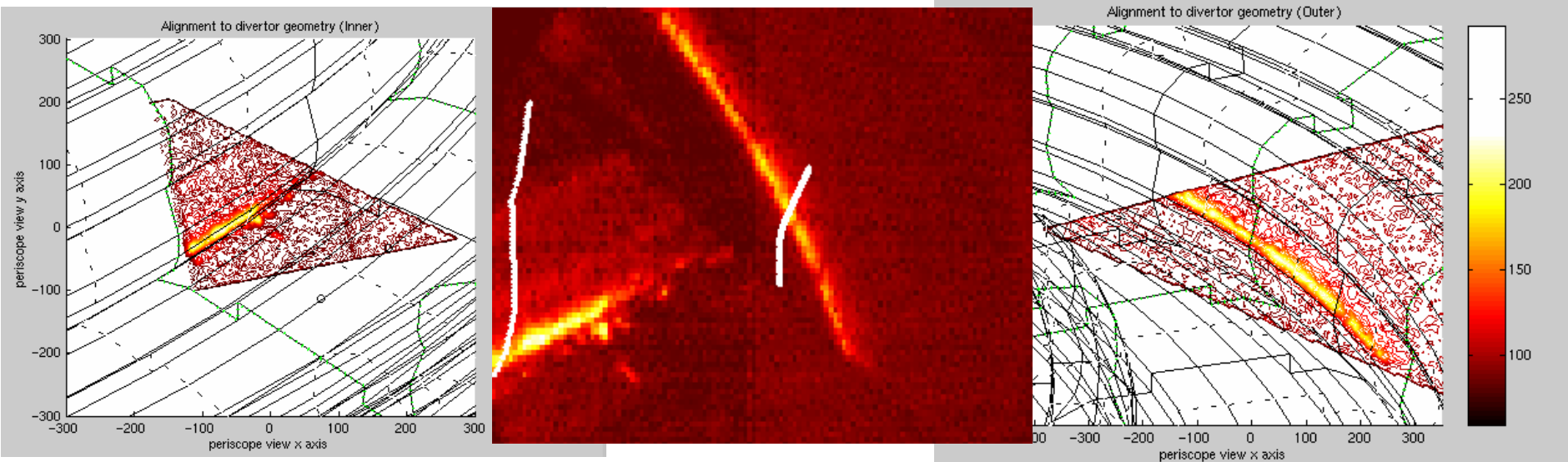
- External magnetic measurements: magnetic probes, flux loops, saddle loops, currents in coils
- Kinetic measurements:  $p_e(R,Z)$  (made of  $n_e$ ,  $T_e$ ,  $T_i$ ,  $Z$ ), including iso-surface information.
- Internal magnetic measurements:  $B_R(R,Z)$ ,  $B_Z(R,Z)$
- MHD information: location of modes
- Strike points, boundary clues

More information:  
position of strike points

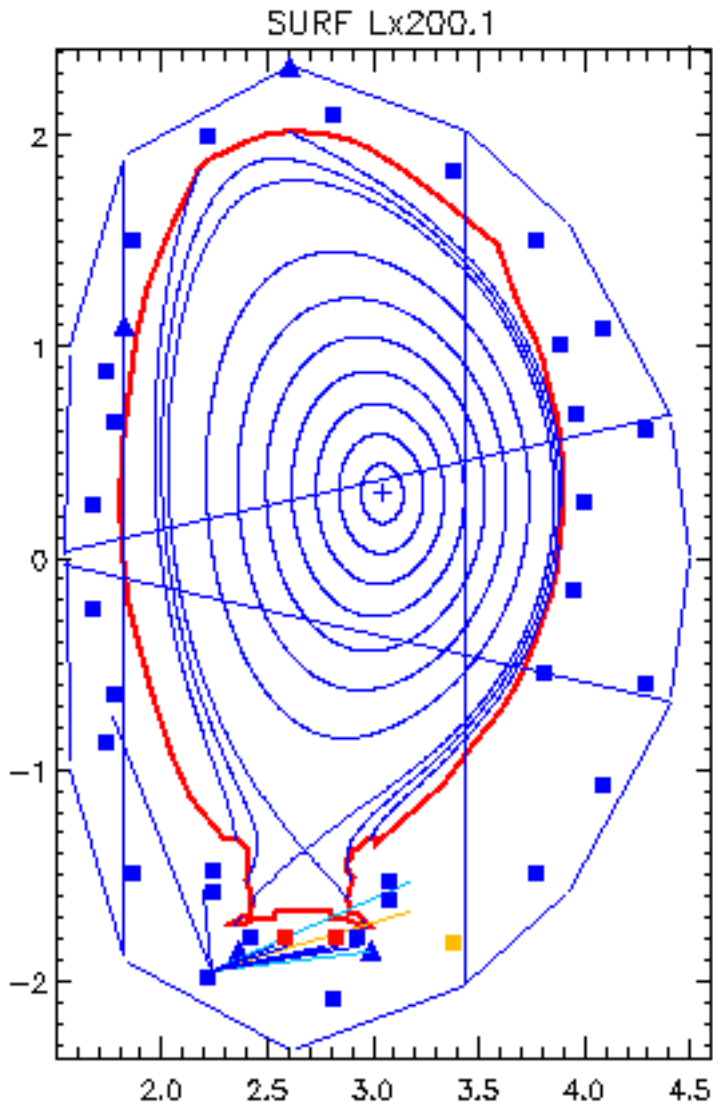
Target Langmuir Probes



## Divertor Infrared Camera



# External magnetic measurements in JET



- magnetic probes, flux loops, saddle loops, currents in coils
- Probes are far apart, saddles produce very integral measurements, there are very few flux loops.  
(to be improved with JET-EP)

# Representation of variables and functions

$$\begin{aligned}\vec{B} &= F\nabla\zeta + \nabla\zeta \times \nabla\Psi \\ &= \vec{B}_{\text{toroidal}} + \vec{B}_{\text{poloidal}}\end{aligned}$$

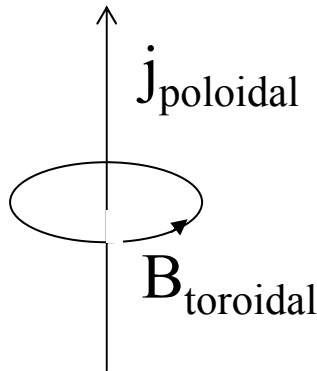
$$\Psi = (\text{Poloidal Flux})/2\pi$$

$$F = RB_{\text{toroidal}}$$

$p$  and  $F$  are flux functions,  $\bar{\Psi}$  is normalised flux

$$p = p(\bar{\Psi}), F = F(\bar{\Psi}), \quad \bar{\Psi} = (\Psi - \Psi_{\text{axis}})/(\Psi_{\text{LCFS}} - \Psi_{\text{axis}})$$

$F(\bar{\Psi})$  is related to the poloidal current density:



$$\oint B_{\text{toroidal}} R d\zeta = \mu_0 \int j_{\text{poloidal}} dS$$

$$\underbrace{2\pi RB_{\text{toroidal}}}_{2\pi F} = \mu_0 I_{\text{poloidal}}$$

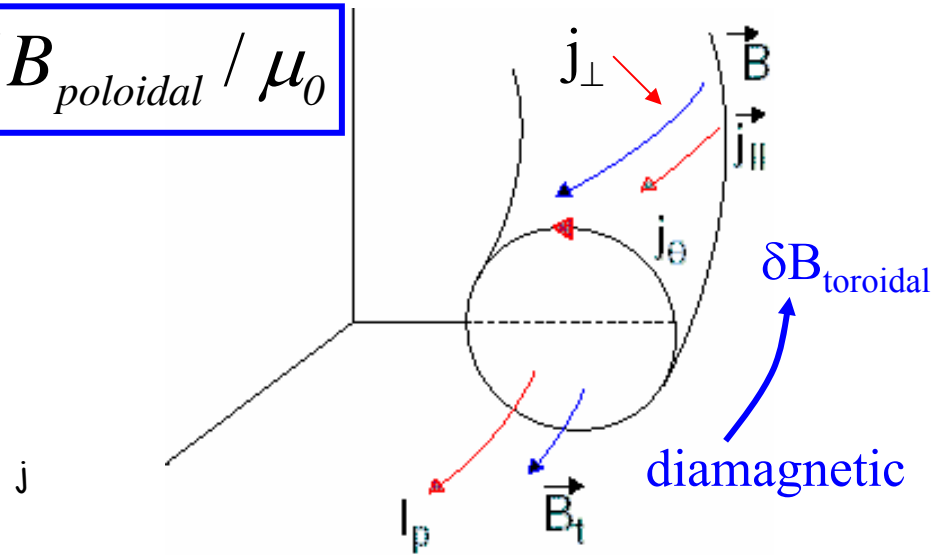


# Para and diamagnetism in tokamak plasma equilibrium

$$\vec{j} \times \vec{B} = \nabla p$$

$$j_{poloidal} = -F' B_{poloidal} / \mu_0$$

$$j_{toroidal} = R p' + \frac{1}{R} \frac{FF'}{\mu_0}$$



Diamagnetism:

$j_{pol}$  from  $j_{perp}$  creates  $\delta B_{toroidal}$  that opposes  $B_{toroidal}$  vacuum

Paramagnetism :

$j_{pol}$  from  $j_{parallel}$  creates  $\delta B_{toroidal}$  that increases  $B_{toroidal}$  vacuum

Note: bootstrap is parallel and paramagnetic in tokamaks!

$\vec{j}_{bootstrap} \times \vec{B} = 0$  , bootstrap alone can not hold  $\nabla p$

# How are the reconstructed equilibria used at JET?

1. Chain1 EFIT output is used by most diagnostics for flux-surface mapping

2. Then

Plasma boundary,  $q$  profile and current density representation are fed to another equilibrium code (HELENA)

Flux surfaces outside of  $x_{\psi}=95\%$  or  $99\%$  are removed (X-point is difficult for codes like HELENA)

The new equilibrium is fed to a transport code, which will re-calculate current profile evolution based on energy and poloidal field diffusion (JETTO, TRANSP)

# Improving equilibrium reconstruction

- Central profile measurements ( $p$ ,  $B_{\text{pol}}$ ) for magnetic axis position and current and pressure peaking.
- Confinement region  $T_e$  for iso-surfaces
- Edge  $p$  for pedestal structure (inboard???)
- Strike points for boundary information
- Pedestal and edge  $B_p$  (MSE, Li beam?) for pedestal  $j$  profile. (Note: MSE is often difficult to use before ELMs, as it is polluted by Er)
- **Stationary plasmas**: large MHD modes introduce poloidal and toroidal asymmetries, hard to deal with.
- MHD modes for  $q$  locations