Dynamics of transport barrier relaxations in tokamak edge plasmas

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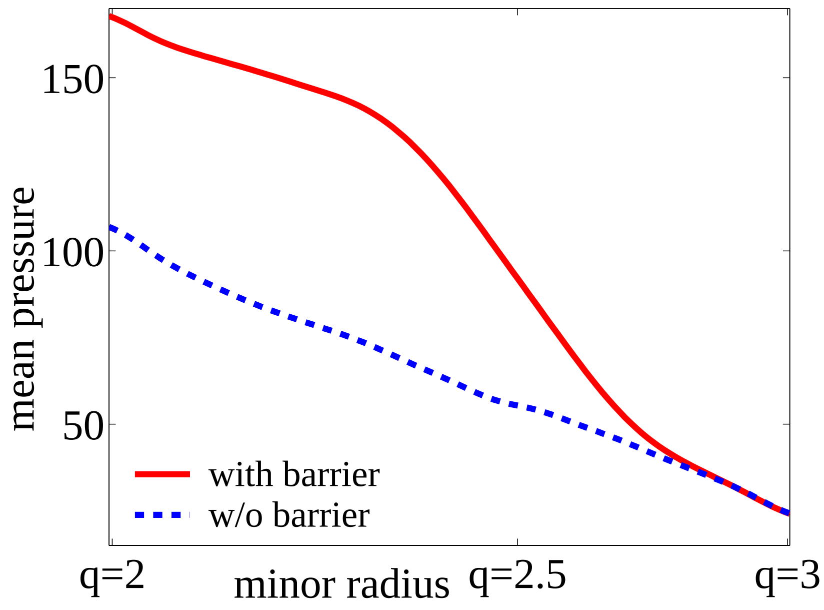
Introduction

- The operational regime of future fusion reactors is characterized by
  - an edge transport barrier,
  - relaxation oscillations of the barrier (Edge Localized Modes, ELMs).



- Explanations for relaxations are usually based on MHD instability,
  - analysis of linear stability properties, no dynamics.
- Most existing dynamical models are phenomenological,
  - not based on 1st principles, i.e. turbulence simulations.
- Frequency, crash time and energy release are central issues.

Outline

- Overview of existing reduced dynamical models for transport barrier relaxations.
- 3D fluid turbulence simulations.
- Subsequent reduced 1D model.
- Systematic reduction $\longrightarrow$ 0D model.

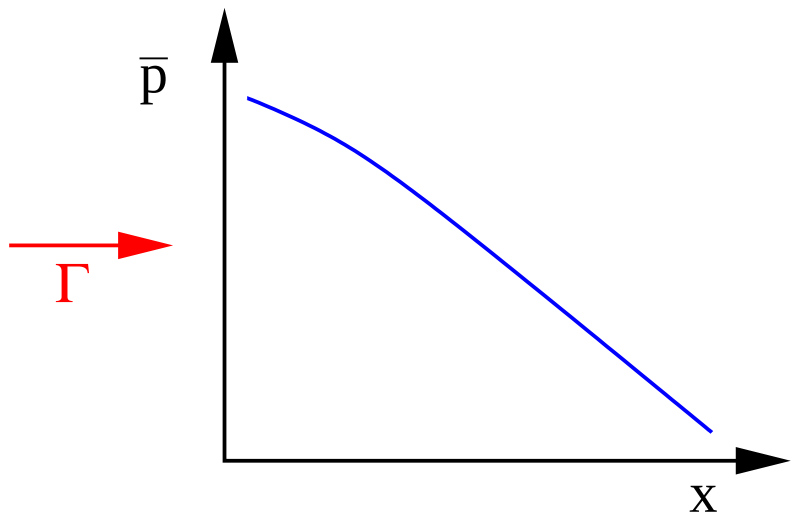
Reduced models for barrier dynamics: not straightforward

- simple model (1D):

  transport eqn. coupled to instability amplitude eqn. at plasma edge

$$\begin{aligned} \partial_t \bar{p} &= -\partial_x \left(\chi_0 \bar{\pi} + \chi_1 |\xi|^2 \bar{\pi} - \Gamma \right) \\ \partial_t \xi &= \gamma_0 (\bar{\pi} - \alpha_c) \xi + \nu_0 \partial_x^2 \xi \end{aligned}$$

$$\bar{\pi} = -\partial_x \bar{p}$$



$\bar{p}$: pressure profile, ξ: perturbation ampl.,

Γ: incoming energy flux, x: minor radius

- no oscillations, stable fixed point, robust property

Possible modifications to obtain oscillations or relaxations

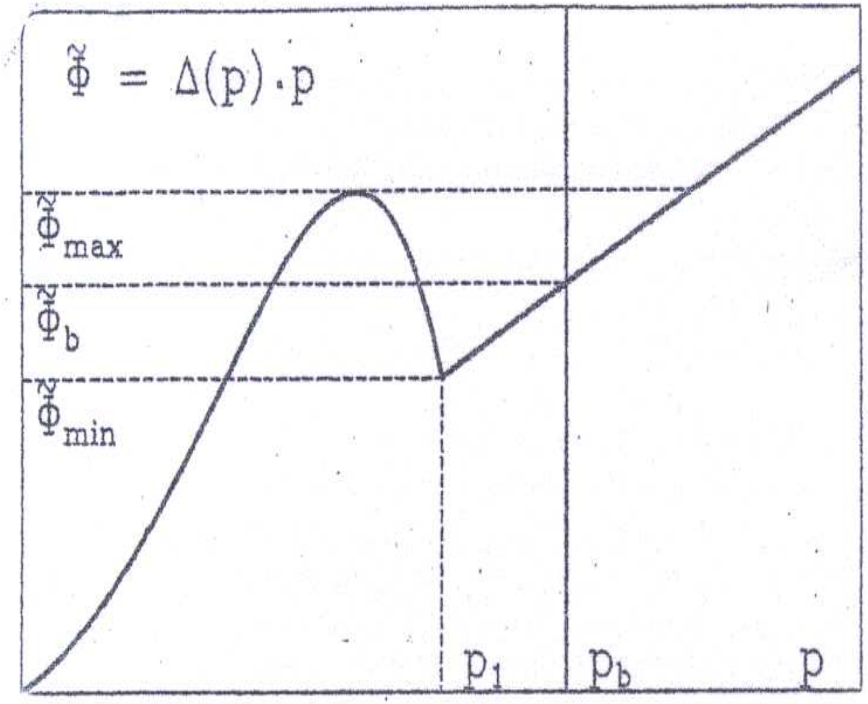
- Introduction of S–curve
  - for dependency of flux vs gradient (due to ExB shear flow),
  - in dynamical eqn. for perturbation amplitude (explosive instability),
  - in dynamical eqn. for ExB shear flow (multiple states: L–H).
- Introduction of characteristics of ideal MHD eigenmodes
  - vanishing growth rate below threshold,
  - radial shape of global modes.

S–curve for flux vs gradient produces relaxations

- Introduce ambient turbulent flux Γ_{turb} due to drift waves, etc.
- $\tilde{\Phi}(\bar{\pi}) = \Gamma_{\text{turb}}(\bar{\pi}) + \chi_0 \bar{\pi}$: S-curve due to turb. stabilization by ExB shear flow.

$$\begin{aligned} \partial_t \bar{p} &= -\partial_x \left[\tilde{\Phi}(\bar{\pi}) + \chi_1 |\xi|^2 \bar{\pi} - \Gamma \right] \\ \partial_t \xi &= \gamma_0 (\bar{\pi} - \alpha_c)\, \xi + \nu_0 \partial_x^2 \xi \end{aligned}$$

$$\bar{\pi} = -\partial_x \bar{p}$$



$p \equiv \bar{\pi}$

Lebedev, Diamond, PoP 95

- Relaxations, frequency ↗ with power.
- More sophisticated models available.

Explosive instability

- Account for non-linear terms in amplitude equation:

  first is destabilizing, second is stabilizing.

$$\begin{aligned} \partial_t \overline{p} &= -\partial_x \left(\chi_0 \overline{\pi} + \chi_1 |\xi|^2 \overline{\pi} - \Gamma \right) \\ \partial_t \xi &= \gamma_0 \left(\overline{\pi} - \alpha_c \right) \xi + \mu \xi^2 - \nu \xi^3 \end{aligned}$$

$$\overline{\pi} = -\partial_x \overline{p}$$

- Dynamics close to Van der Pol oscillations. Cowley, Wilson, PPCF 03

Multiple states for shear flow

- L–H transition: multiple states for ExB shear flow $\overline{u}(\overline{\pi})$,
  and: effective flux $\chi_{\text{eff}}(\overline{u})\,\overline{\pi}$ depends on shear flow.

$$\begin{aligned} \partial_t \overline{p} &= -\partial_x \left[\chi_{\text{eff}}(\overline{u})\,\overline{\pi} - \Gamma\right] \\ \partial_t \overline{u} &= -(\overline{\pi} - \alpha_c) - \mu_1 \overline{u}^3 + \mu_2 \overline{u} + \nu \partial_x^2 \overline{u} \end{aligned}$$

$$\overline{\pi} = -\partial_x \overline{p}$$

- Ginzburg–Landau type, limit cycle oscillations.
- No perturbation amplitude, more appropriate for “dithering”.
- Generalization to ELMs available. Itoh, Itoh, PRL 91, PRL 95

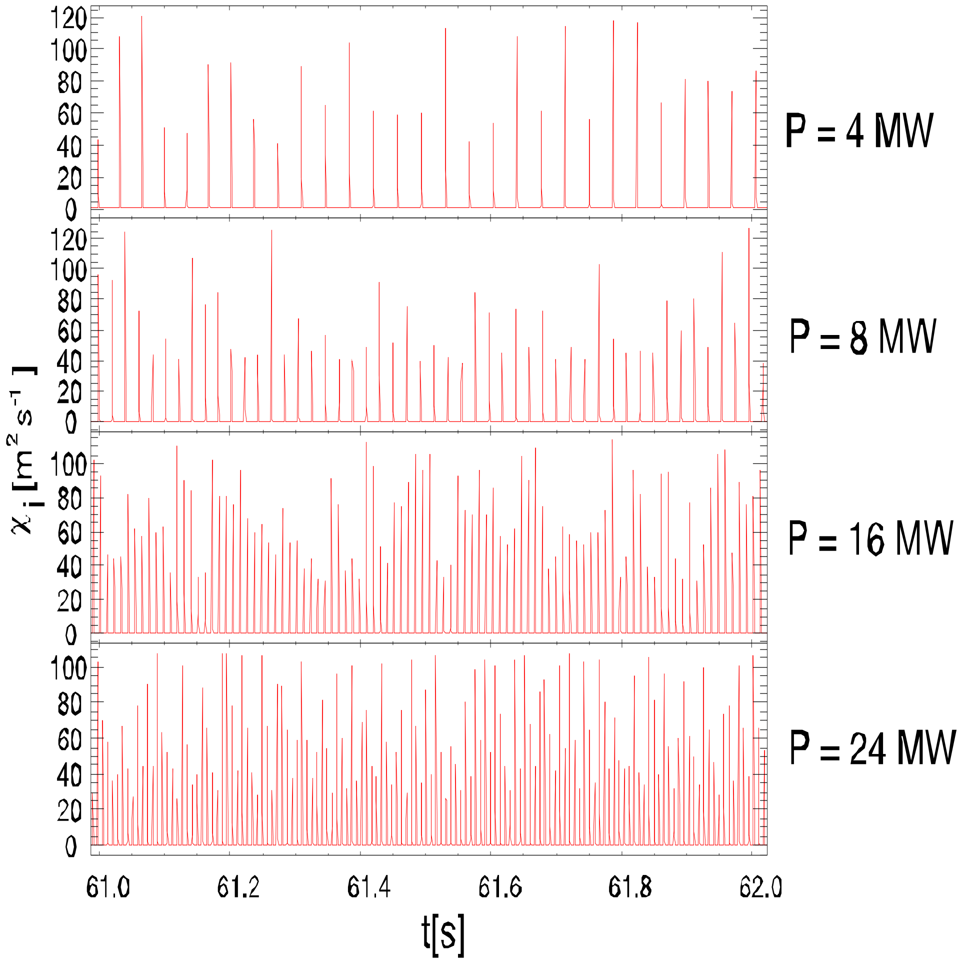
Linear ideal MHD instability model

- Linear ideal MHD eigenmodes:
  - growth rate ≈ 0 below threshold,
  - global mode structure.
- Modeled by
  - Heaviside funct. H on growth rate,
  - Gaussian shape G in eff. diffusivity

$$\partial_t \xi = \gamma_0 \left(\bar{\pi} - \alpha_c\right) H\left(\bar{\pi} - \alpha_c\right) \xi - \nu\left(\xi - \xi_0\right)$$

\+ transp. code with $\chi_{\text{eff}} \propto |\xi|^2 G(x)$

- Relaxations, frequency ↗ with power.
- More sophisticated models (peeling).



Lönnroth, Parail, PPCF 04

Bécoulet, Huysmans, EPS 03

State of the art

- Most existing models are phenomenological.
- Difficult to reproduce relaxations with frequency ↘ with power.
- Turbulence simulations of relaxations exist, based on turbulent ExB flow generation, no barrier.
- Need for 1st principles based model, i.e. 3D turbulence simulations, reproducing i) transport barrier ii) complete relaxation cycle.

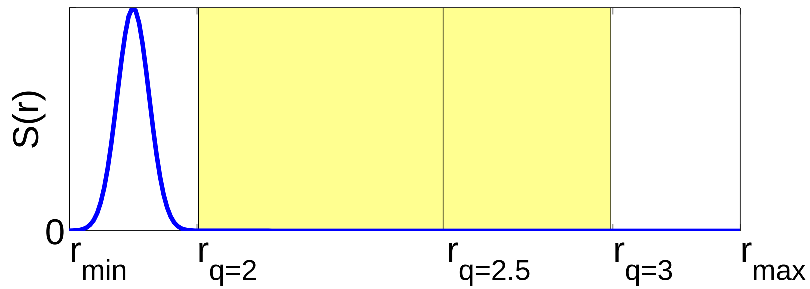
3D edge turbulence simulations with transport barrier

- turbulence model: resistive ball. modes, reduced MHD equations

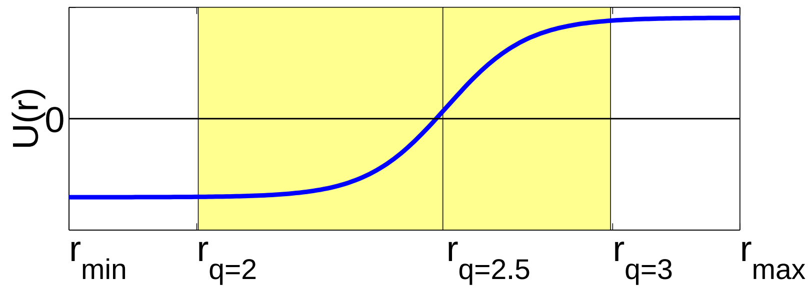
$$\partial_t \nabla_\perp^2 \phi + \left\{ \phi, \nabla_\perp^2 \phi \right\} = -\nabla_\parallel^2 \phi - \mathbf{G} p + \nu \nabla_\perp^4 \phi$$

$$\partial_t p + \{\phi, p\} = \delta_c \mathbf{G} \phi + \chi_\parallel \nabla_\parallel^2 p + \chi_\perp \nabla_\perp^2 p + S$$

- 3D toroidal geometry at plasma edge
- driven by incoming flux $\Gamma_{\text{in}} = \int_{r_{\text{min}}}^{r} S \, \mathrm{d}r'$, press. profile evolves self-consistently
- barrier generated by imposed flow U, locally sheared, $\omega_{E\,\text{ext}} = (\partial_r U)_{\text{max}}$

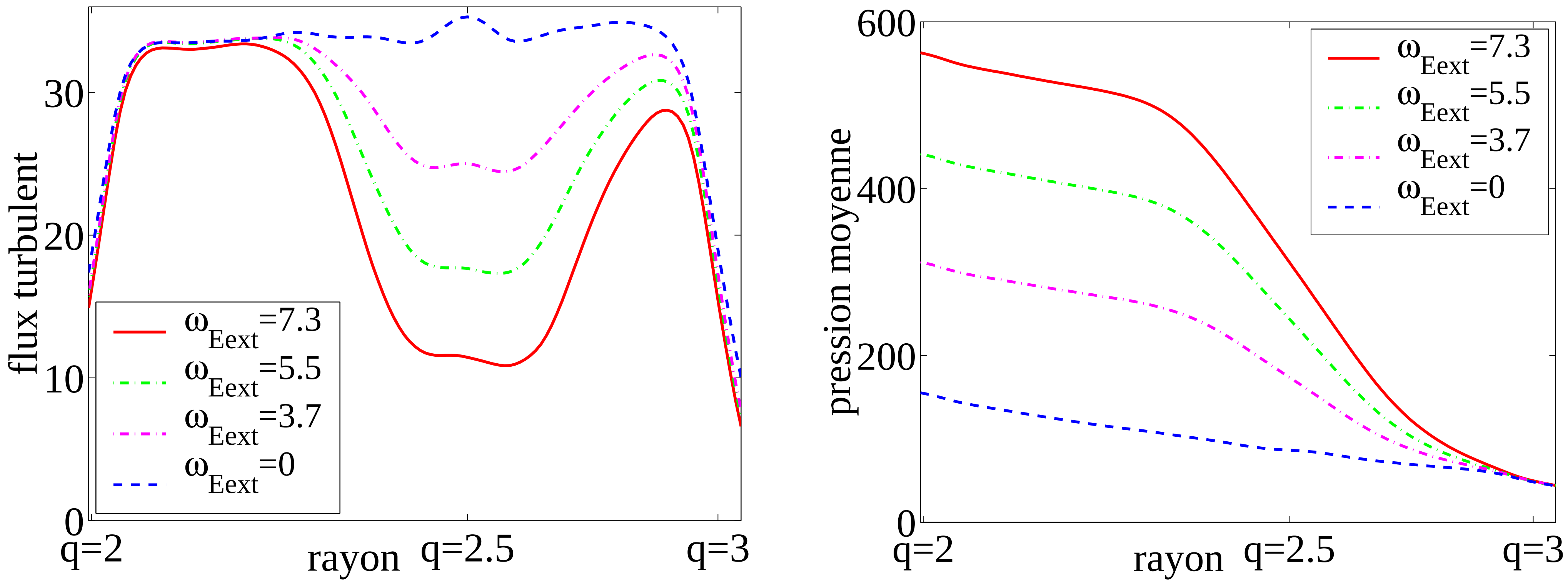


radial profile of source S



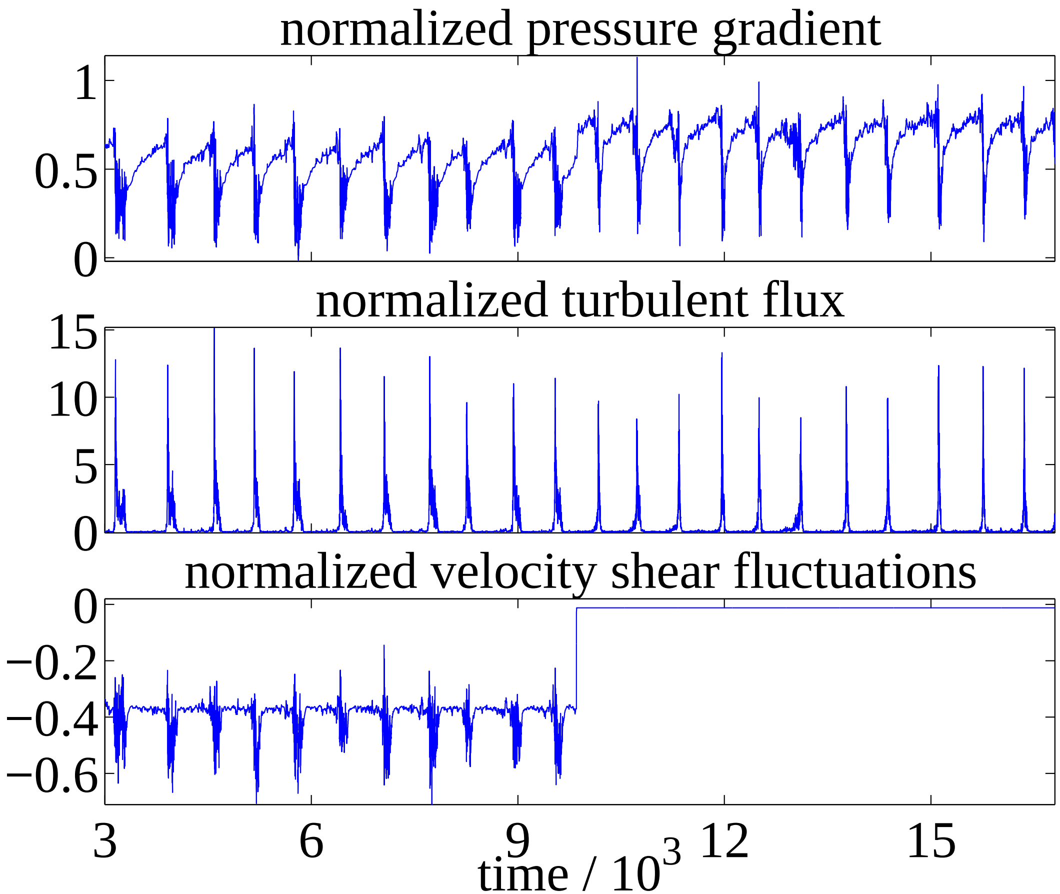
radial profile of imposed flow U

Strong local ExB shear → formation of barrier



turbulent flux profile | time averaged | pressure profile

Barrier relaxation oscillations appear



1. $|\partial_x \overline{p}| \,/\, (\Gamma_{in}/\chi_\perp)$

2. $\Gamma_{turb}/\Gamma_{in}$

3. $(\omega_E - \omega_{E\,\text{ext}}) \,/\, \omega_{E\,\text{ext}}$
   [fixed to 0 for $t \geq 10^4$]

- all evaluated at barrier center

- observed in a range of Γ_{in}, $\omega_{E\,\text{ext}}$
- robust property

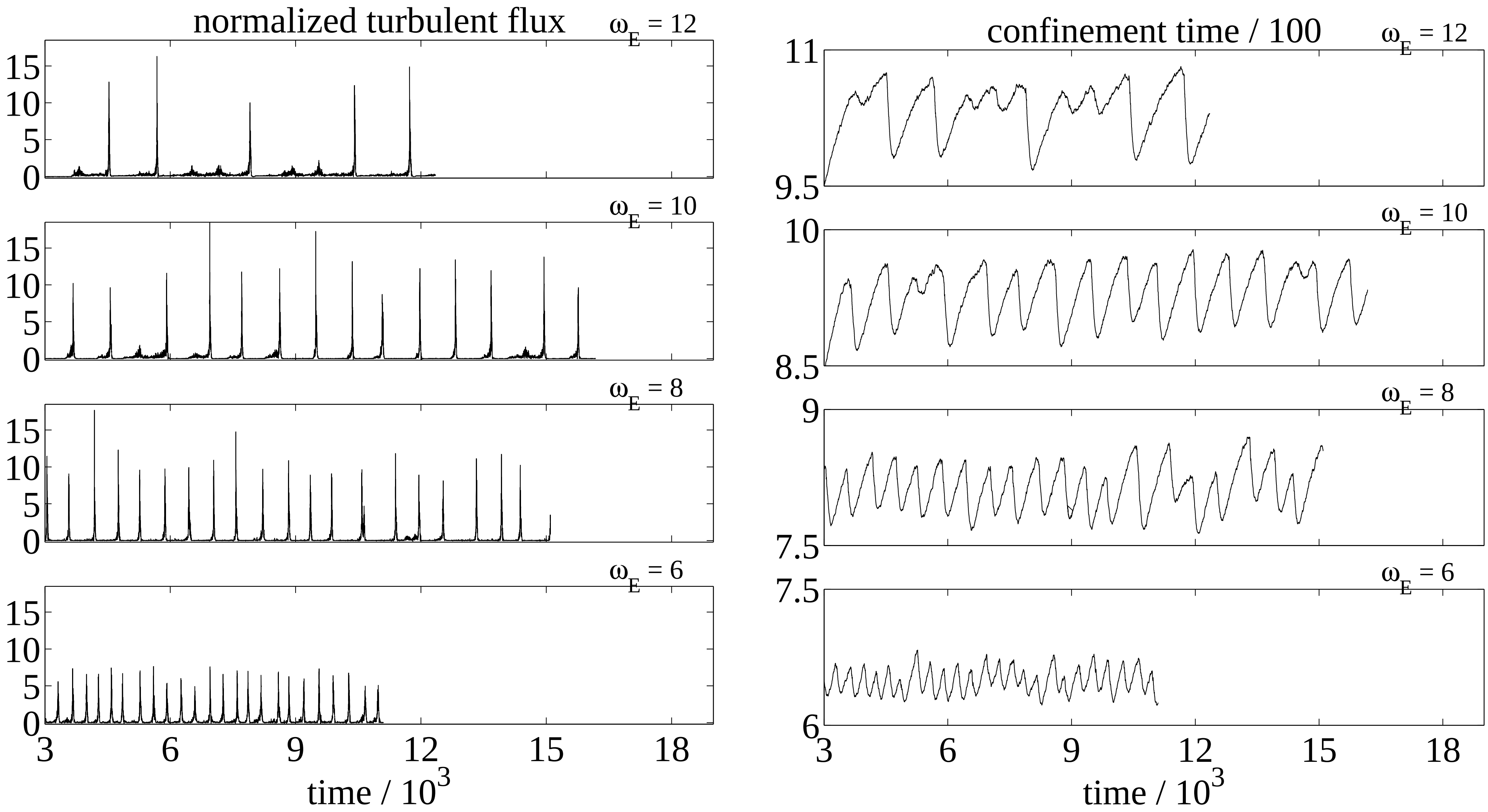
scenario:

turb. state $\rightarrow$ relaxations $\rightarrow$ quiescent st.

$\Longrightarrow \omega_E$

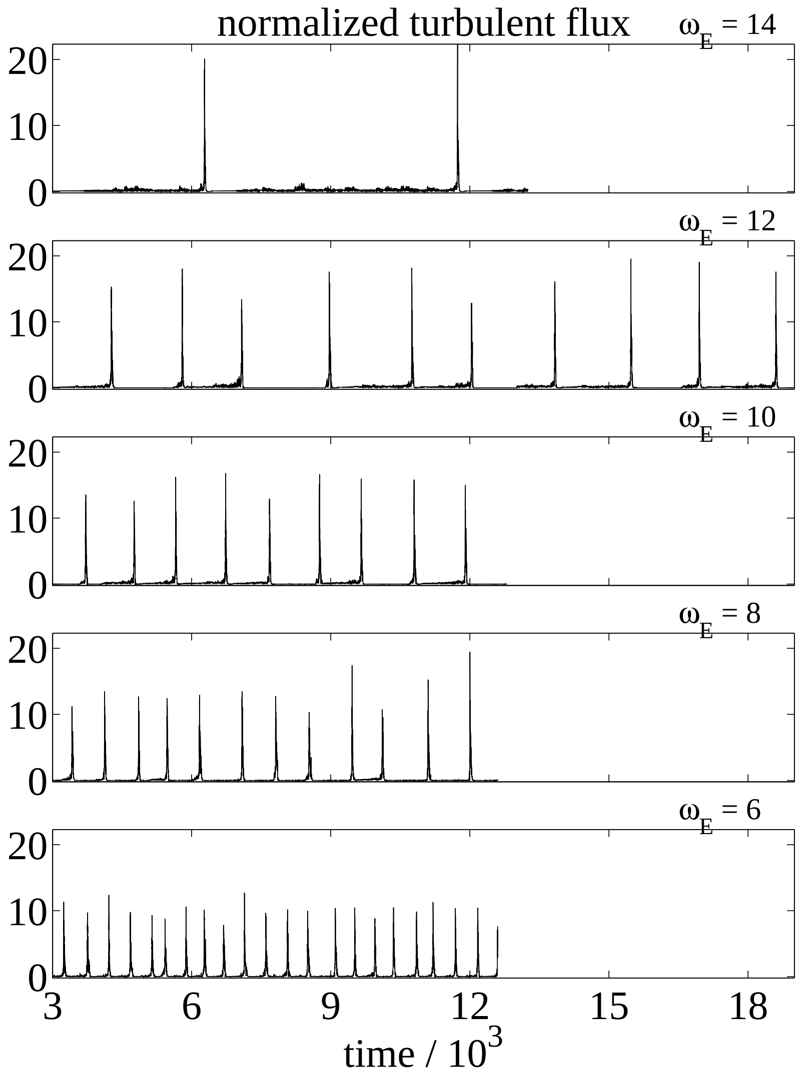
no barrier $\rightarrow$ barrier

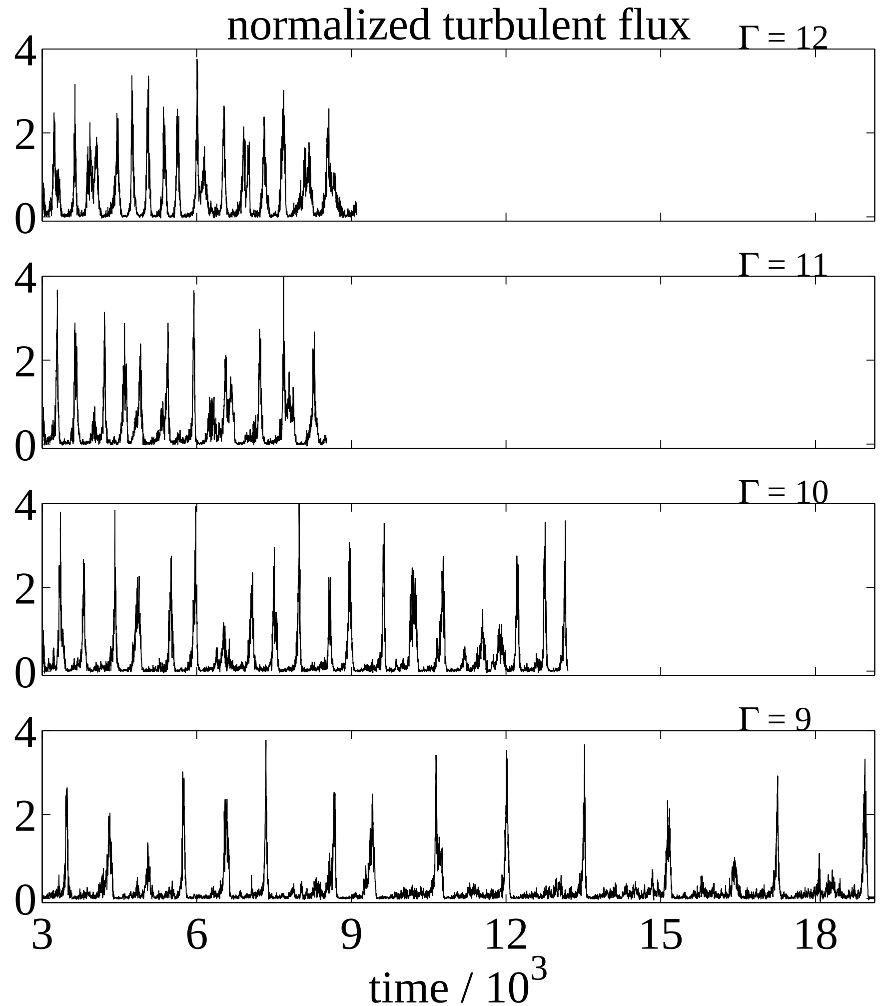
Fixed input power: frequency decreases with shear



input power: $\Gamma_{in} = 36$, shear layer width: $0.1 L_x$

Fixed power: freq. ↘ shear ; fixed shear : freq. ↗ power

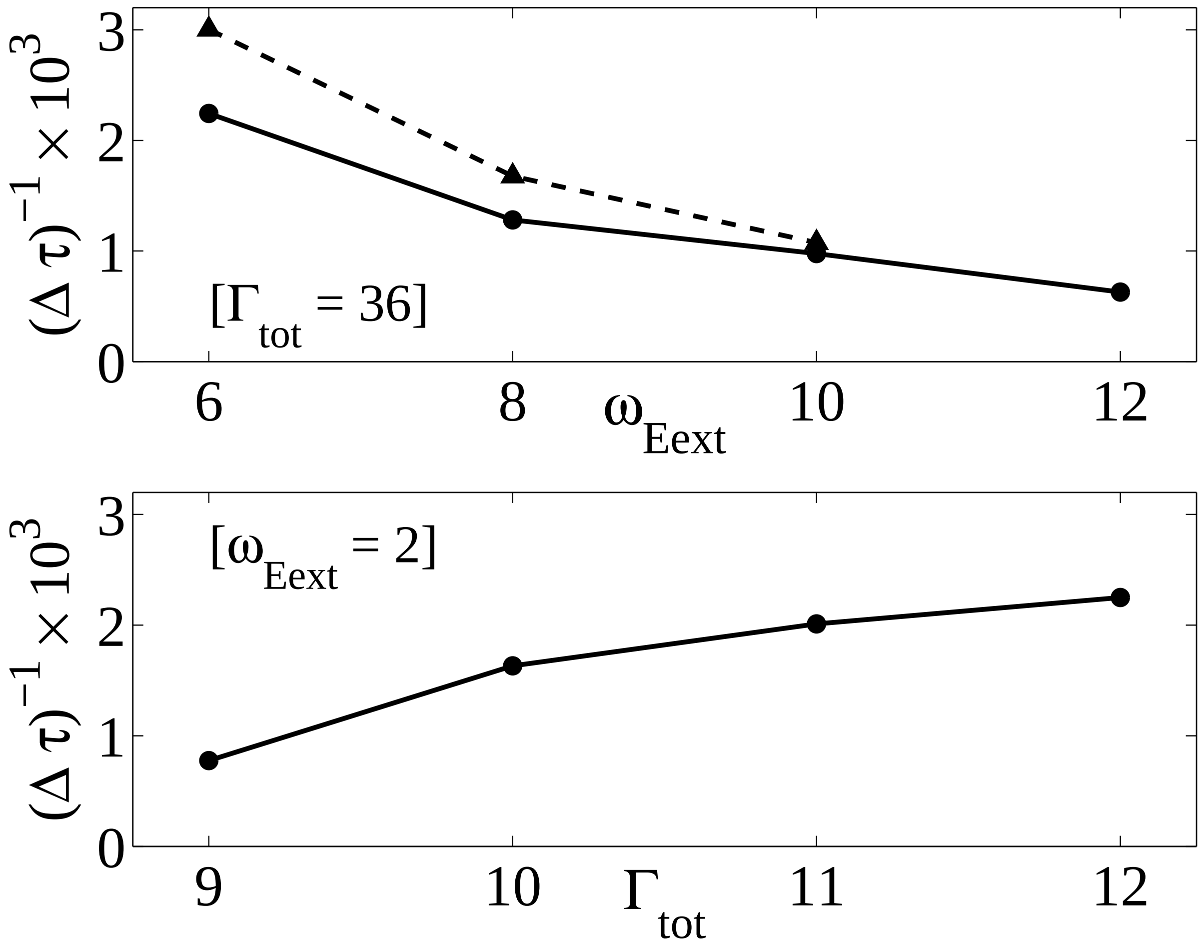




input power: $\Gamma_{in} = 36$ shear layer width: $0.12L_x$ flow shear: $\omega_E = 2$

Frequency dependence: two opposite trends

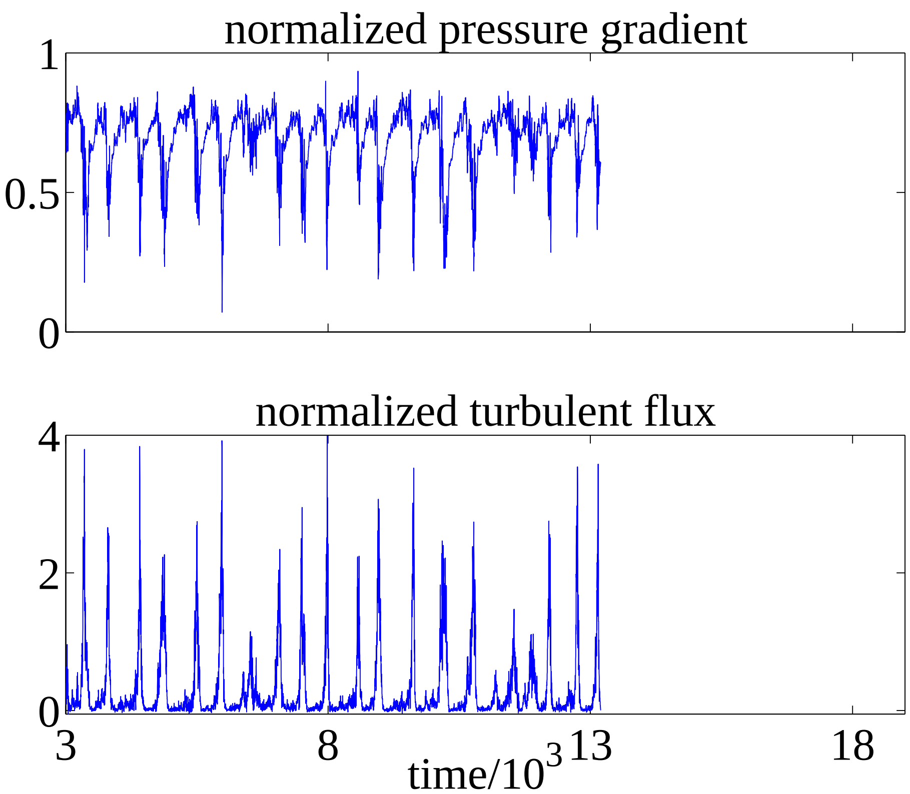
inverse averaged time lag between relaxations



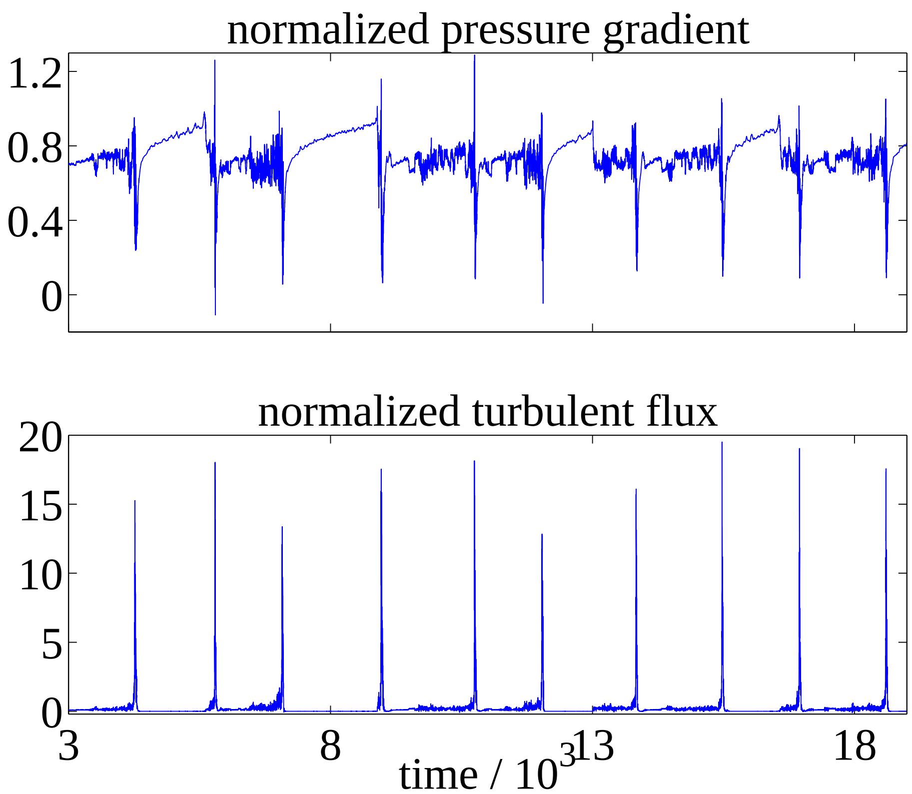
shear layer width: $0.12L_x$ (–), $0.1L_x$ (- -)

Frequency dependence: two opposite trends

if ω_E increases fast enough with Γ_{in} $\longrightarrow$ frequency decreases with Γ_{in}.

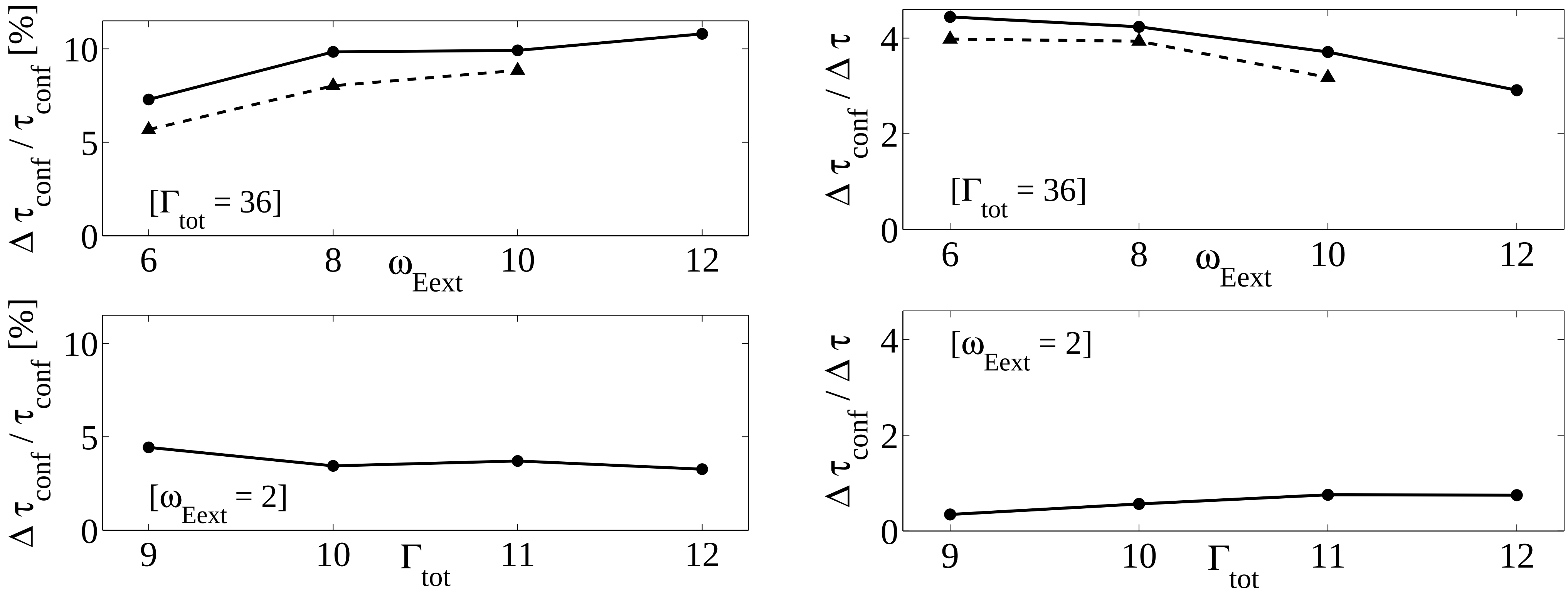


$\Gamma_{\text{in}} = 10\ , \quad \omega_E = 2$



$\Gamma_{\text{in}} = 36\ , \quad \omega_E = 12$

Relative drop of confinement time: ↗ shear



relative averaged drop of conf. time

“frequency” × averaged energy loss

Possible relaxation mechanisms excluded

- Relaxations persist even if ExB shear flow is frozen

  $\longrightarrow$ mechanism $\neq$ turbulent shear flow generation.

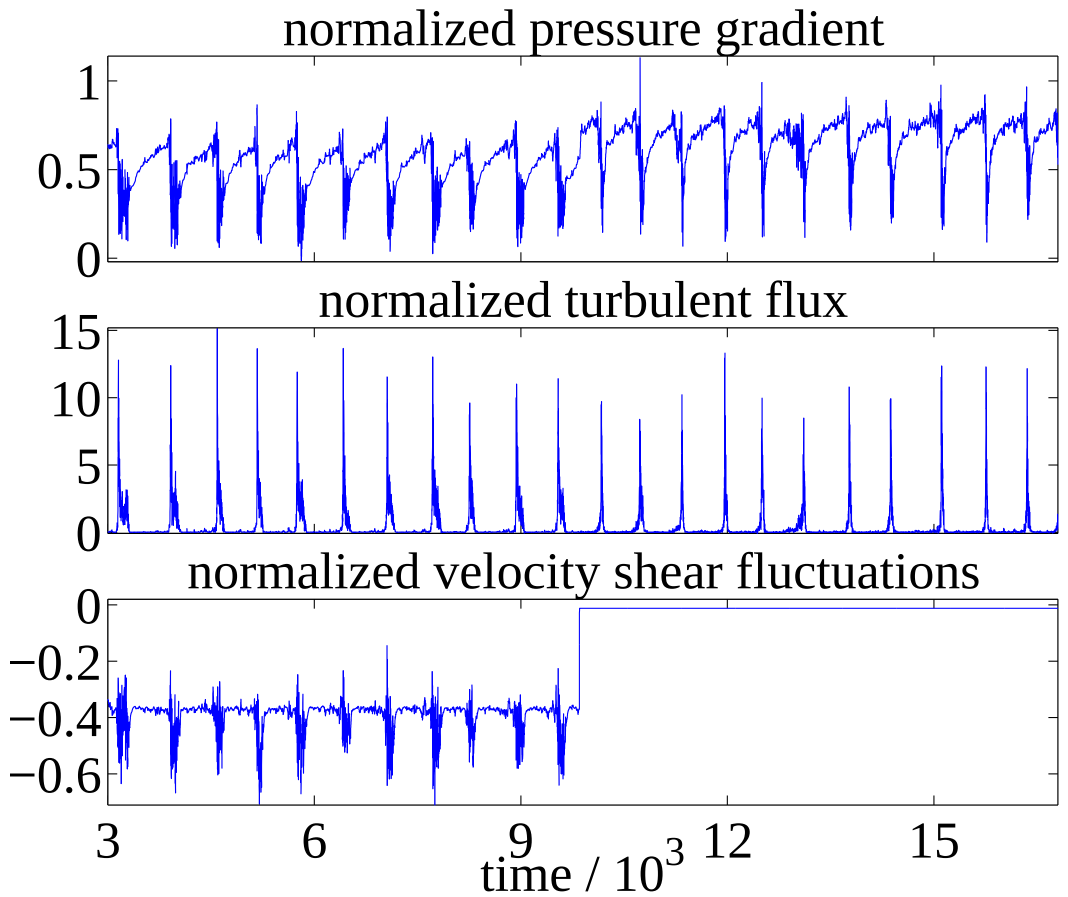
- No significant variation of modes localized outside barrier

  $\longrightarrow$ mechanism $\neq$ toroidal mode coupling.

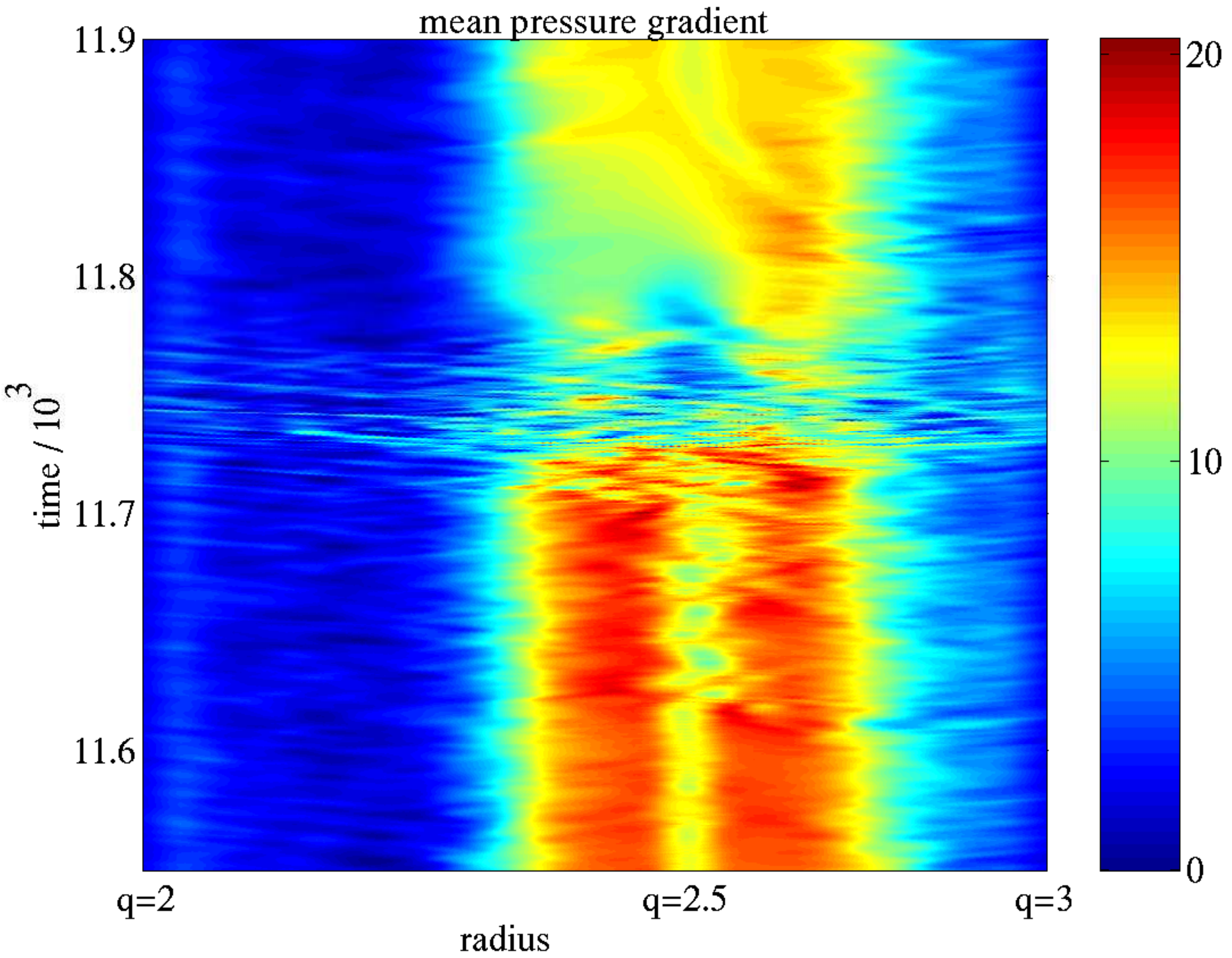
- All fluctuations die out when suppressing curvature

  $\longrightarrow$ Kelvin–Helmholtz stable.

$\Gamma_{\text{in}} = 36\,, \quad \omega_E = 8$

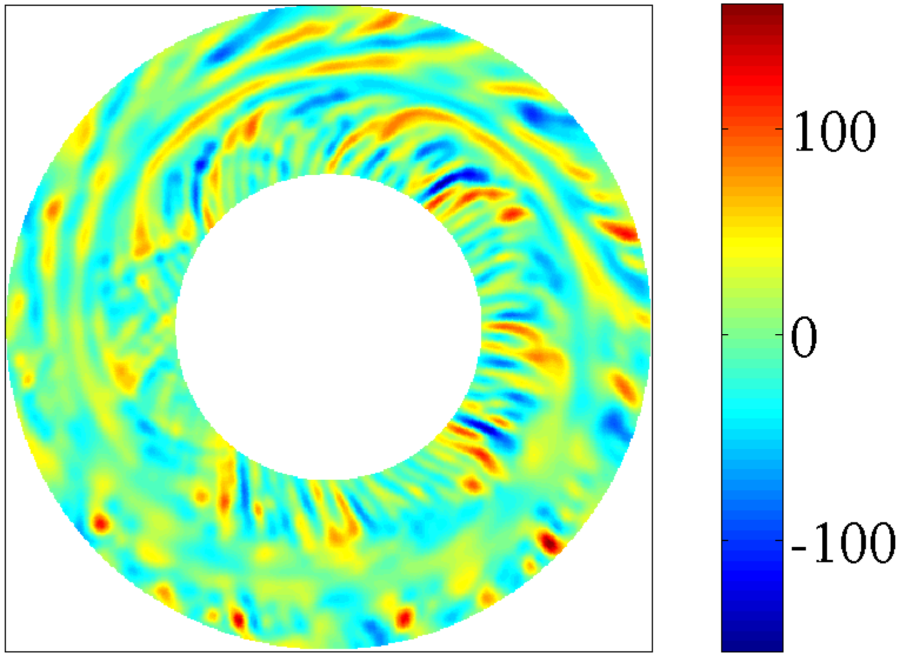


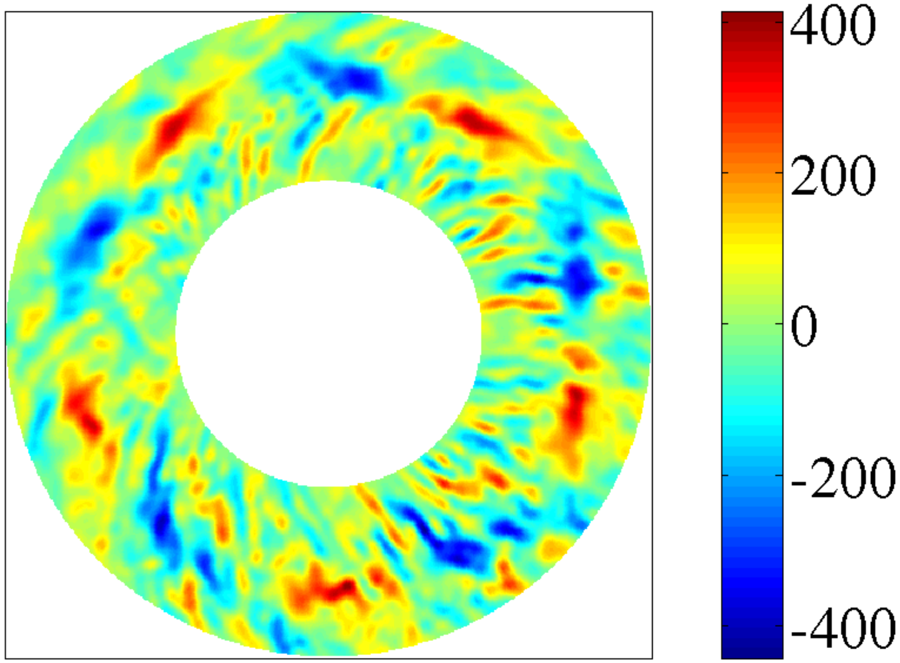
Relaxation event propagates radially away from barrier center



mean pressure gradient versus radius and time

Relaxation governed by mode at barrier center



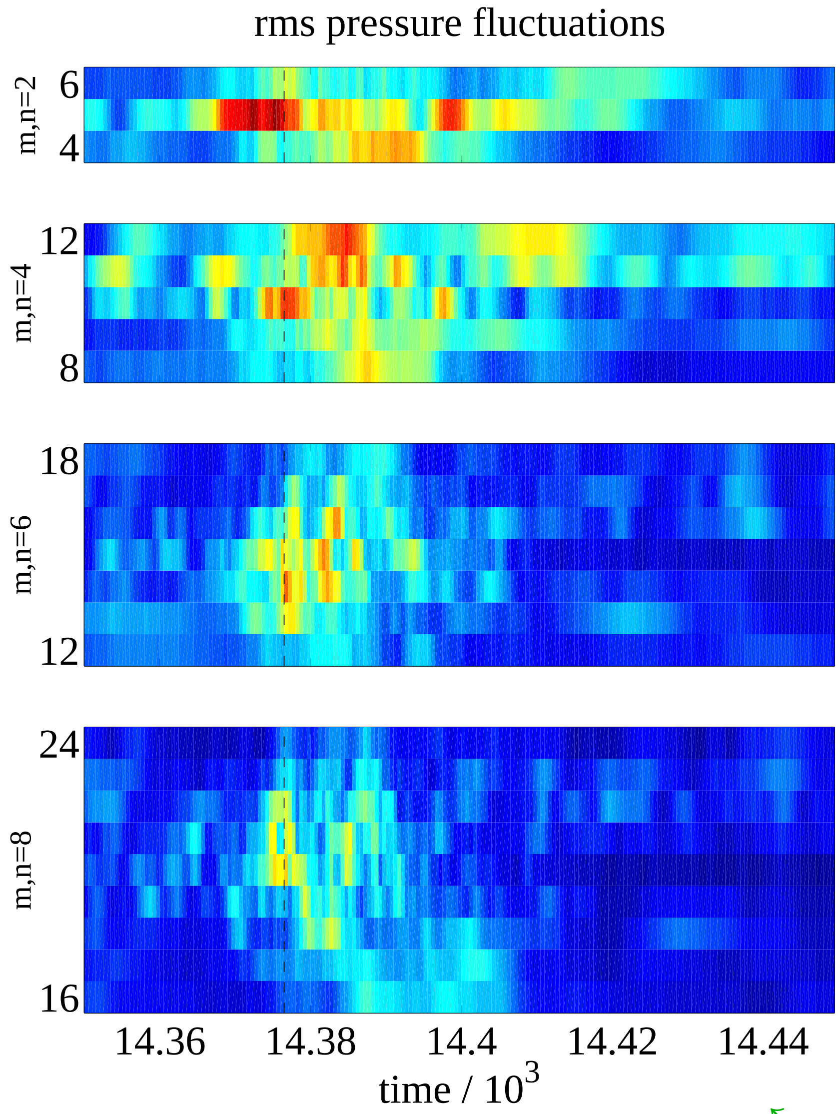


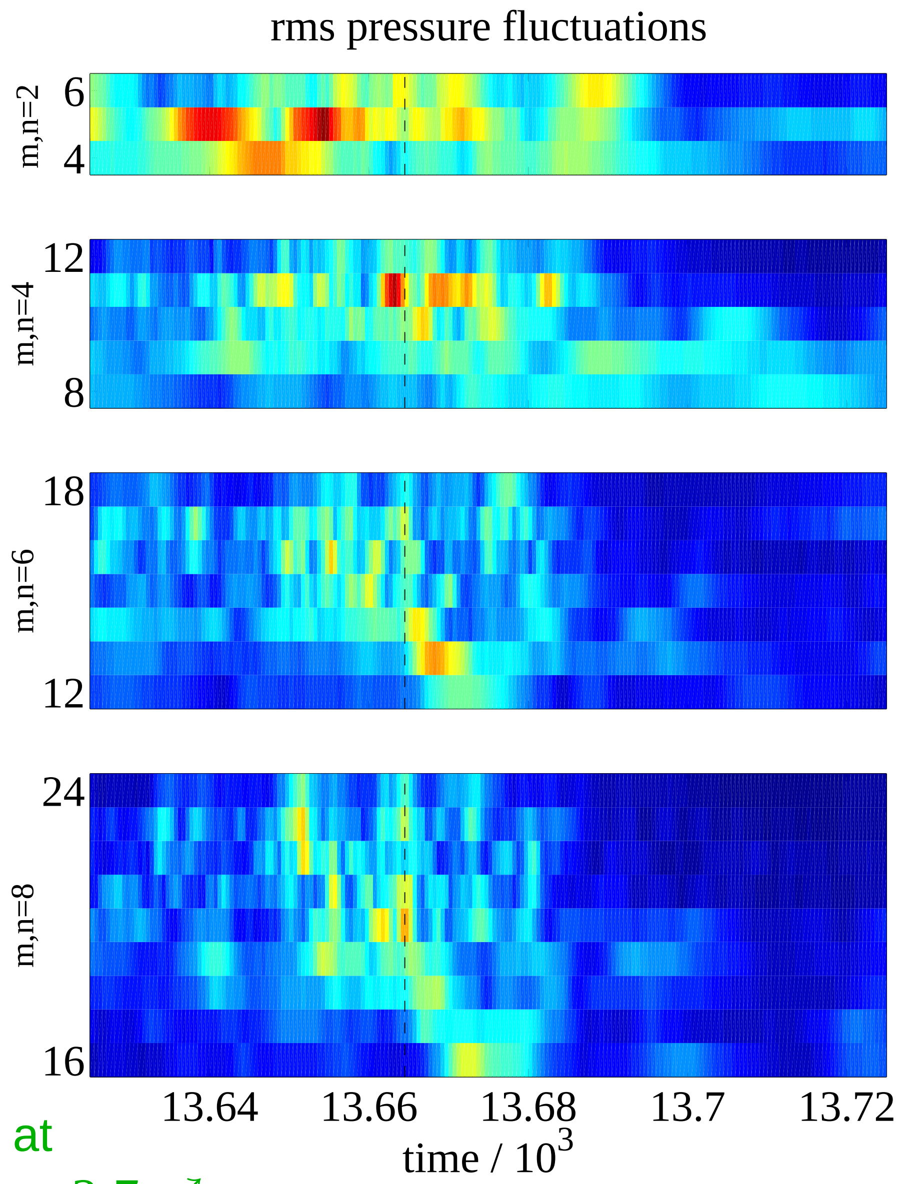
snapshots of potential fluctuations

between relaxations

during relaxation

Relaxation governed by mode at barrier center

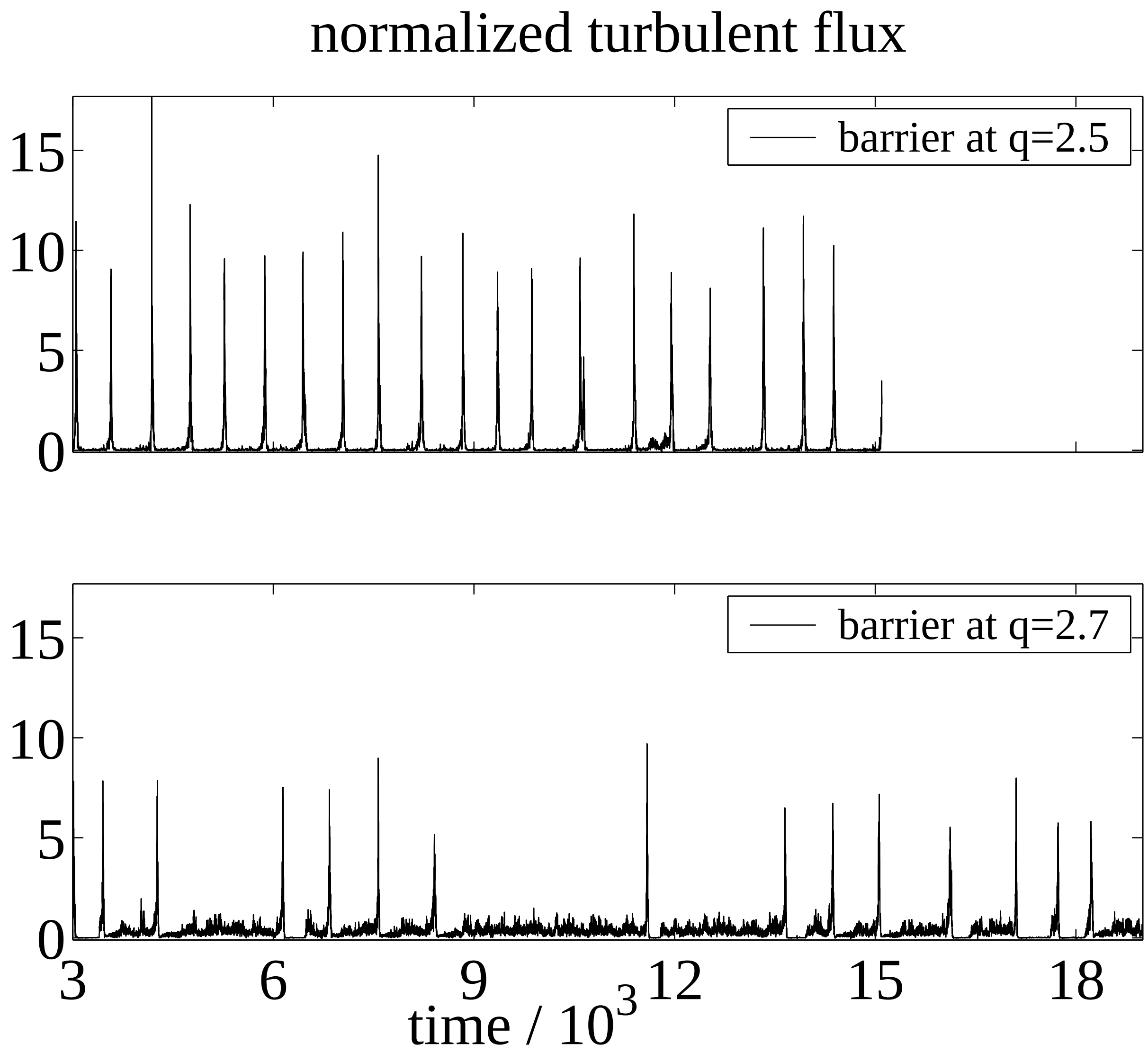




barrier at

↖ $q = 2.5$ $q = 2.7$ ↗

Less regular relaxation for barrier at $q = 2.7$



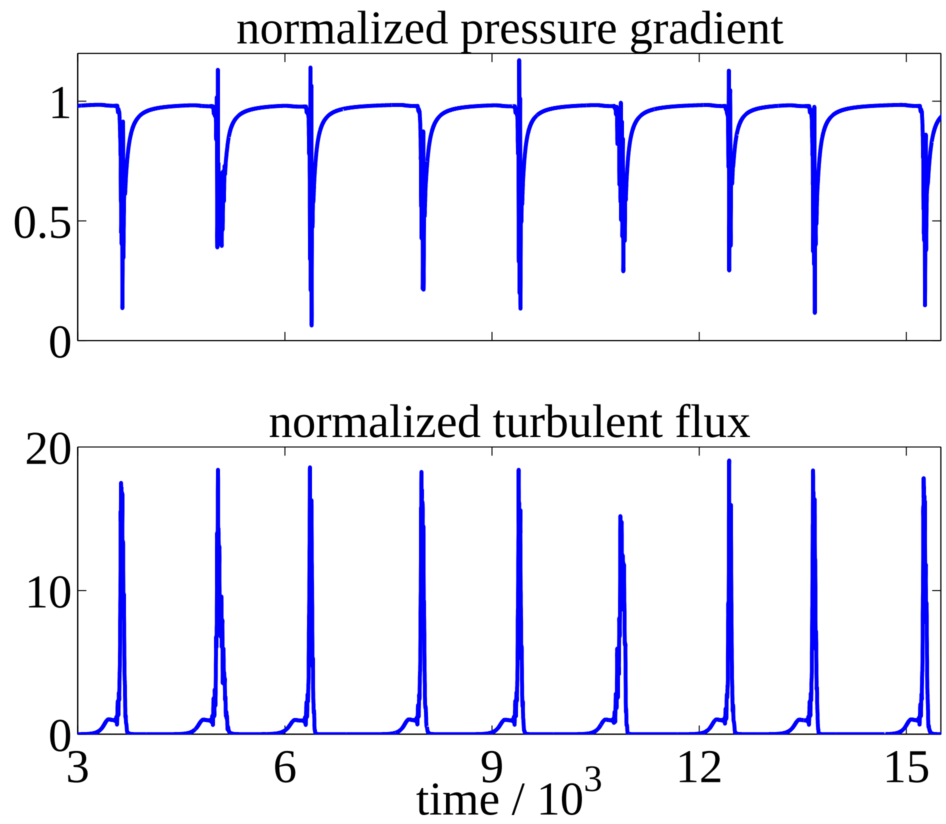
1D model for central mode amplitude $\tilde{p}(x,t)$ & profile $\overline{p}(x,t)$

$$\partial_t \overline{p} = -2\gamma_0 \partial_x |\tilde{p}|^2 + \chi_\perp \partial_x^2 \overline{p} + S$$
$$\partial_t \tilde{p} = \gamma_0 (-\partial_x \overline{p} - \alpha_0)\, \tilde{p} - \mathrm{i}\omega'_E x \tilde{p} - \chi'_{\|} x^2 \tilde{p} + \chi_\perp \partial_x^2 \tilde{p}$$

$x = r - r_0$: radial dist. from barrier center

m: poloidal wavenumber of central mode

- reproduces relaxation oscillations
- ExB shear $\omega'_E = \omega_E m / r_0$
  $\longrightarrow$ nonlinear short-term dynamics.
- Not described by linear modes
  (long-term dynamics).



Description by linear modes is not appropriate

For $\partial_x \overline{p} = -\alpha$, evolution equation for $\tilde{p}$ is linear:

$$\partial_t \tilde{p} = \gamma_0 (\alpha - \alpha_0)\, \tilde{p} - i\omega_E' x \tilde{p} - \chi_\parallel' x^2 \tilde{p} + \chi_\perp \partial_x^2 \tilde{p}$$

- most unstable eigenmode:

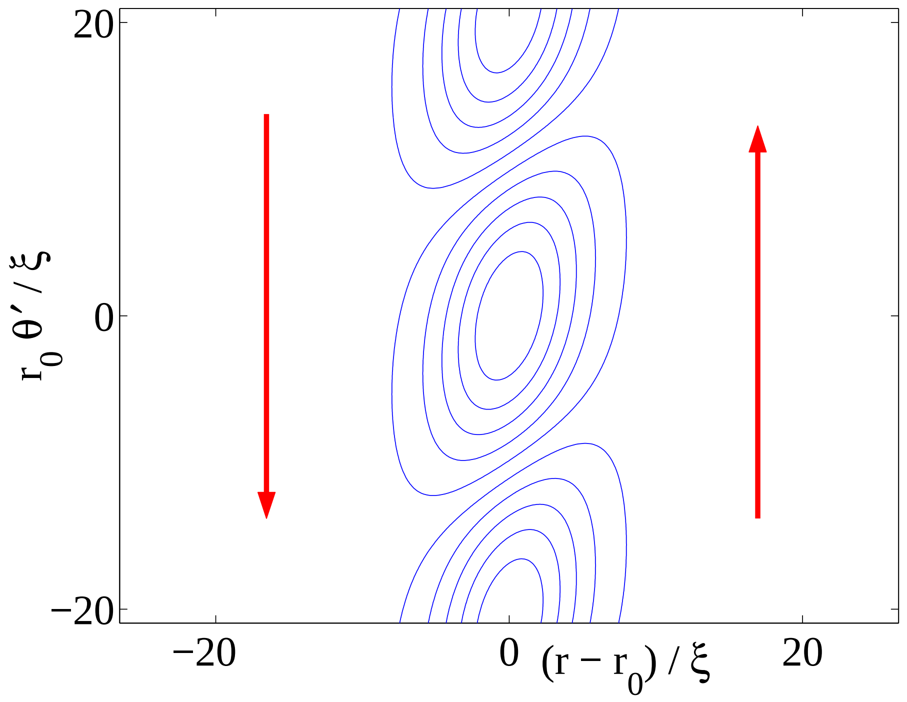
$$\tilde{p} e^{im\theta} \sim \exp\left(-\frac{x^2}{2\sigma^2} + im\theta - \frac{i\omega_E' x}{2\sqrt{\chi_\parallel' \chi_\perp}} \right)$$

  with $\sigma^2 = \sqrt{\chi_\perp / \chi_\parallel'}$

- growth rate:

$$\gamma = \gamma_0 (\alpha - \alpha_0) - \frac{\omega_E'^2}{4\chi_\parallel'} - \sqrt{\chi_\perp \chi_\parallel'}$$

- when $i\omega_E' x$ term replaced by shift of instability threshold $\longrightarrow$ no oscillations



Time delay in stabilization by ExB shear flow

- Short term dynamics of initial pulse

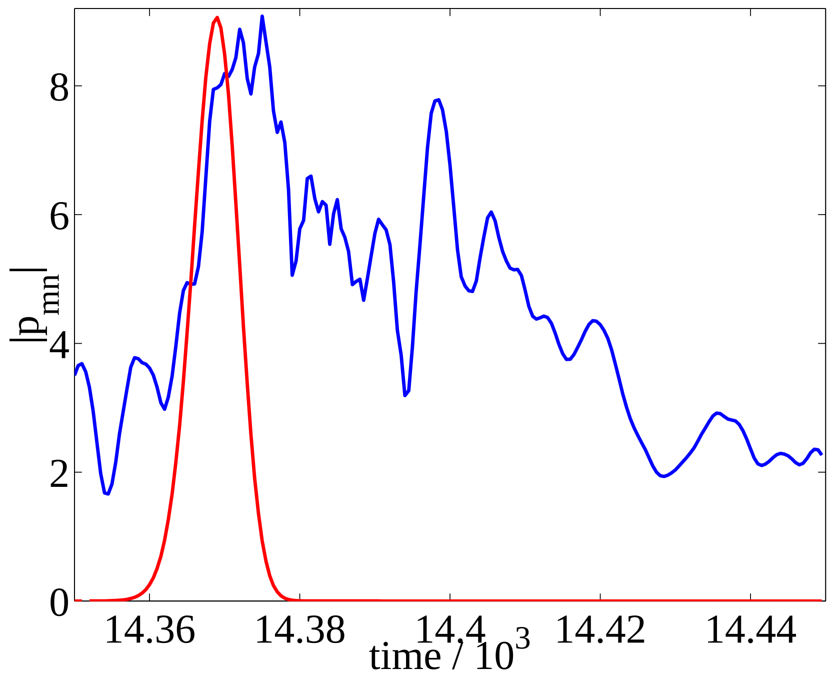
$$\tilde{p}\,(x,t=0) \propto \delta(x)$$

with $-\partial_x \bar{p} = \alpha$ and $\chi'_{\|}$ term neglected.

- Solution : $$\tilde{p} \propto \exp\left[\gamma'_0 t - t^3/(3\tau_D^3)\right]$$

$$\gamma'_0 = \gamma_0\,(\alpha - \alpha_0),\; \tau_D = \left(\tfrac{1}{4}\chi_\perp \omega_E'^2\right)^{-1/3}.$$

- Transient growth before stabilization.
- τ_D large for small $\chi_\perp$ (barrier) and low m.
- Clearly observed in simulations (— curve).



0D model reproduces oscillations
Radial mode structure $\neq$ linear mode

- From 1D model $\rightarrow$ relevant radial structures: $\tilde{p}_R(x)$, $\tilde{p}_I(x)$, $\bar{p}_0(x)$
  [linear mode in case $\partial_x \bar{p} = const$: $\tilde{p}_{\text{lin}} = \tilde{p}_R + \mathrm{i}\tilde{p}_I$]

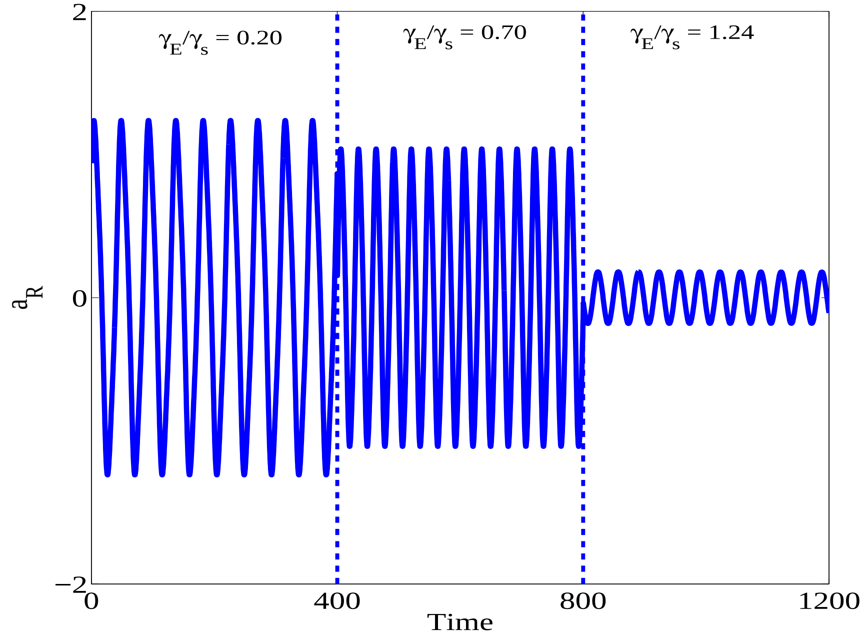
- Projection: $\tilde{p}(x,t) = a_R p_R + \mathrm{i} a_I p_I$, $\bar{p}(x,t) = -\alpha x + a_0 \bar{p}_0$

- Amplitude equations:

$$
\begin{aligned}
\dot{a}_R &= (\Gamma - \delta_1 a_0)\, a_R + \Omega_1 \,(a_R - a_I) \\
\dot{a}_I &= (\Gamma - \delta_2 a_0)\, a_I - \Omega_2 \,(a_I - a_R) \\
\dot{a}_0 &= -\gamma_0 a_0 + 2\delta_1 a_R^2 + 2\delta_2 a_I^2
\end{aligned}
$$

$$\Gamma = \gamma_0 (\alpha - \alpha_0) - \gamma_s - \gamma_E \qquad \gamma_s^2 = \chi'_{\parallel}\chi_{\perp}$$

$$\Omega_1 \approx \Omega_2 \approx 2\gamma_E e^{-\gamma_E/\gamma_s} \qquad \gamma_E = {\omega'_E}^2/(4\chi'_{\parallel})$$



- same frequency dependence on ExB shear as in 3D simulations

Conclusions

- 3D nonlinear turbulence simulations based on 1st principles show
  - onset of transport barrier,
  - barrier relaxation oscillations.
- Mechanism based on effective time delay for stabilization by ExB shear, no obvious S–curve, no global mode.
- Mean features are reminiscent of type III ELMs:
  - frequency dependence,
  - resistive ballooning mode model (low temperature plasma),
  - sensitivity to shear flow.

turb. state $\longrightarrow$ relaxations $\longrightarrow$ quiescent st.

$\Longrightarrow \omega_E$

no barrier $\longrightarrow$ barrier