Closure of equilibrium–transport set / Data flow

Grad-Shafranov eq. provides $\rho(\mathbf{r}),\ I(\rho,t),\ V'(\rho)$, flux average metric coefficients.

$$\Delta^*\psi = r^2 \mathrm{div}\frac{\nabla\psi}{r^2} = -4\pi^2\left(\mu_0 r^2 \frac{\partial p}{\partial\psi} + I\frac{\partial I}{\partial\psi}\right) \qquad \Longleftarrow \qquad j_\varphi = \left(-\nabla p + j_\vartheta B_\varphi\right)/B_\vartheta$$

and requires $p',\ II'$ as functions of ψ.

Transport equations

$$4\pi^2\mu_0 B_0 \sigma_\parallel \left.\frac{\partial\psi}{\partial t}\right|_\Phi = \frac{I^2}{\rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{I}\left\langle\left(\frac{\nabla\rho}{r}\right)^2\right\rangle\frac{\partial\psi}{\partial\rho}\right] \qquad \Longleftarrow \qquad \sigma_\parallel E_\parallel = j_\parallel$$

$$\left.\frac{\partial\left(V' n_e\right)}{\partial t}\right|_\Phi = \frac{\partial}{\partial\rho}\left[V'\left\langle(\nabla\rho)^2\right\rangle D\frac{\partial n_e}{\partial\rho} + \cdots\right] = V' S_e$$

provide $p(\rho),\ \psi(\rho)$ but not II' as required.

We use: $$j_\parallel = -\frac{2\pi}{B_0}\left[I\frac{\partial p}{\partial\psi} + \left(I^2\left\langle\frac{R_0^2}{r^2}\right\rangle + \rho^2\mu^2 B_0^2\left\langle\frac{(\nabla\rho)^2}{r^2}\right\rangle\right)\frac{1}{\mu_0}\frac{\partial I}{\partial\psi}\right]$$

Problems of the closure

On substitution the **Grad-Shafranov** equation becomes

$$\Delta^*\psi = \frac{2\pi\mu_0 B_0}{\rho\mu}\left(\frac{R_0^2 I^2}{\langle B^2\rangle} - \frac{r^2}{R_0^2}\right)\frac{\partial p}{\partial \rho} + \frac{2\pi\mu_0 B_0 I}{\langle B^2\rangle} j_\parallel$$

where $\langle B^2\rangle = I^2\langle 1/r^2\rangle + \rho^2\mu^2\langle(\nabla\rho/r)^2\rangle$

1. GS and transport equations use different grids
   - Mapping required
2. Time delay
   - Iterations?

t	$\Delta^*\psi = \cdots$
$\downarrow$	$\frac{\partial\psi}{\partial t} = \cdots$
$t + \Delta t$	$\Delta^*\psi = \cdots$