A theory based criterion for Internal Transport Barrier formation

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Introduction

- The main purpose of this work is to provide a first principle description of the generation of the Internal Transport Barriers (ITBs).
- Our main result is the derivation of a criterion for the ITB formation which properly captures several experimentally observed features of the transport barrier.

Introduction

- Experimentally observed features:

1. Appearance of ITB at low order rational surface;
2. Zero/Low shear helps;
3. ρ_* at the barrier exceeds a threshold.

Outline

- Basic assumptions and theoretical model.
- Overview of the rigorous results (with very little math!).
- Heuristic derivation of the criterion for the ITB formation.

Basic Assumptions

This is a list of the main assumptions leading to the model:

- In the ITB the turbulence is quenched by velocity shear. This results in lower perpendicular transport;
- Such a sheared velocity is nonlinearly generated and sustained by the turbulence itself (i.e. it is a zonal flow), once a certain condition is met;
- This condition can be properly identified only within an electromagnetic model, since the interaction between perpendicular drift-waves and parallel Alfven dynamics plays a crucial role.

Model

- The electromagnetic fluid equations that we solve represent the simplest physical system compatible with our assumptions:

$$\frac{\partial U}{\partial t} + V_{E\times B} \cdot \nabla U = \nabla_{//} J$$

$$\frac{\partial n}{\partial t} + V_{E\times B} \cdot \nabla n = \rho^2 \nabla_{//} J$$

$$\frac{\partial \psi}{\partial t} - \nabla_{//} \phi = \nabla_{//} n + d_e^2 \left(\frac{\partial J}{\partial t} + V_{E\times B} \cdot \nabla J \right)$$

$$J = -\nabla^2 \psi$$

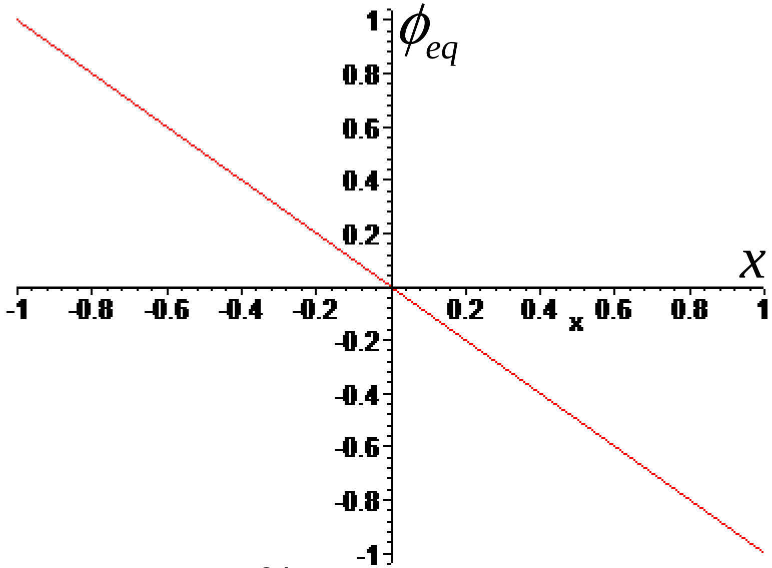
$$U = \nabla \phi$$

- We assume T_i=0, T_e=const, small but finite β, negligible dissipation. The time is normalized to the Alfven time and the length to a macroscopic size.
- d_e is the electron skin depth and ρ is the ion sound Larmor radius.

Mathematical procedure

- We have used a a parametric instability approach to study the zonal flows:

1. Linearize the system around the equilibrium;
2. Evaluate the linear waves/***primary instabilities*** (i.e. dispersion relation and eigenmodes);
3. Linearize the system around the original equilibrium <u>plus</u> one of the linear waves;
4. Evaluate the ***secondary instabilities*** (i.e. the dispersion relation and the eigenmodes of the zonal perturbations)



$$\phi = \phi_{eq} + \tilde{\phi}_{primary}$$
$$\tilde{\phi}_{primary} = \Phi_0 e^{i(\vec{k}\cdot\vec{x} - \Omega_0 t)} + c.c.$$

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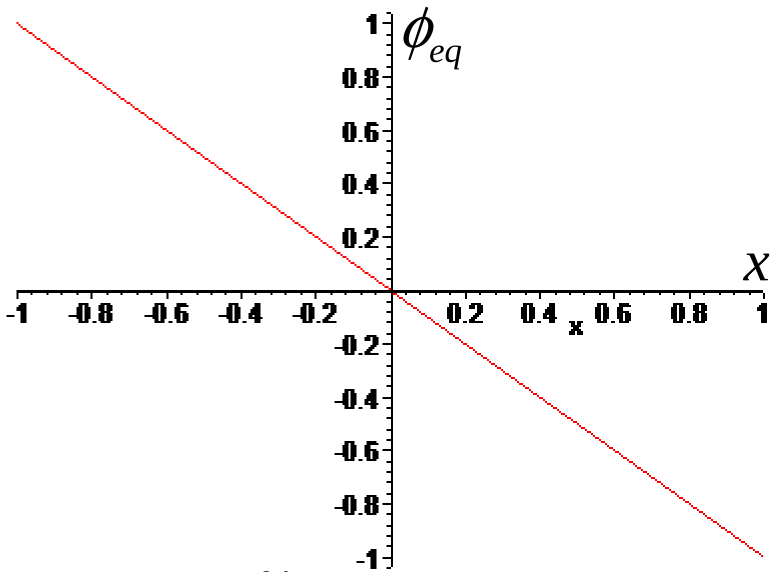

$$\phi = \phi_{eq} + \tilde{\phi}_{primary}$$

$$\tilde{\phi}_{primary} = \Phi_0 e^{i(\vec{k}\cdot\vec{x} - \Omega_0 t)} + c.c.$$

$$\Omega_0 = \Omega_0(parameters, \vec{k}, equilibrium)$$

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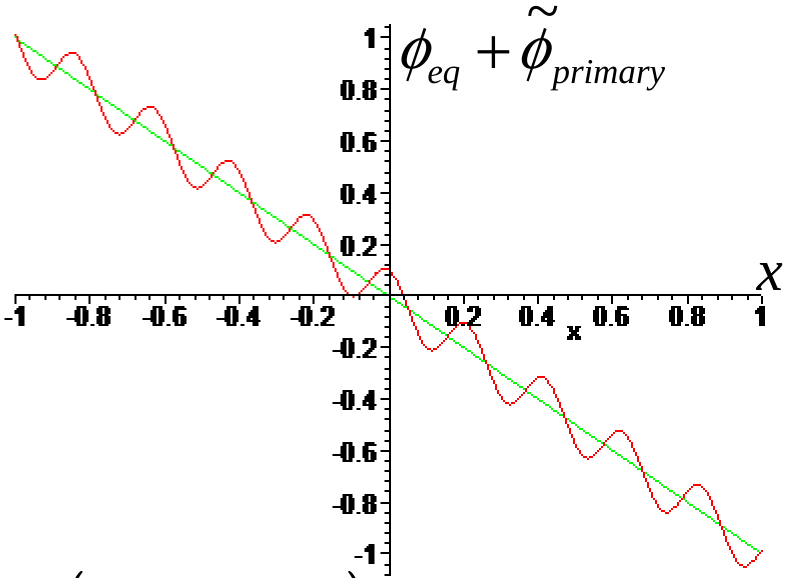


$$\phi = \left(\phi_{eq} + \tilde{\phi}_{primary}\right) + \tilde{\phi}_{secondary}$$

$$\tilde{\phi}_{secondary} = \phi_s e^{i(K_x x - \Omega t)} + $$

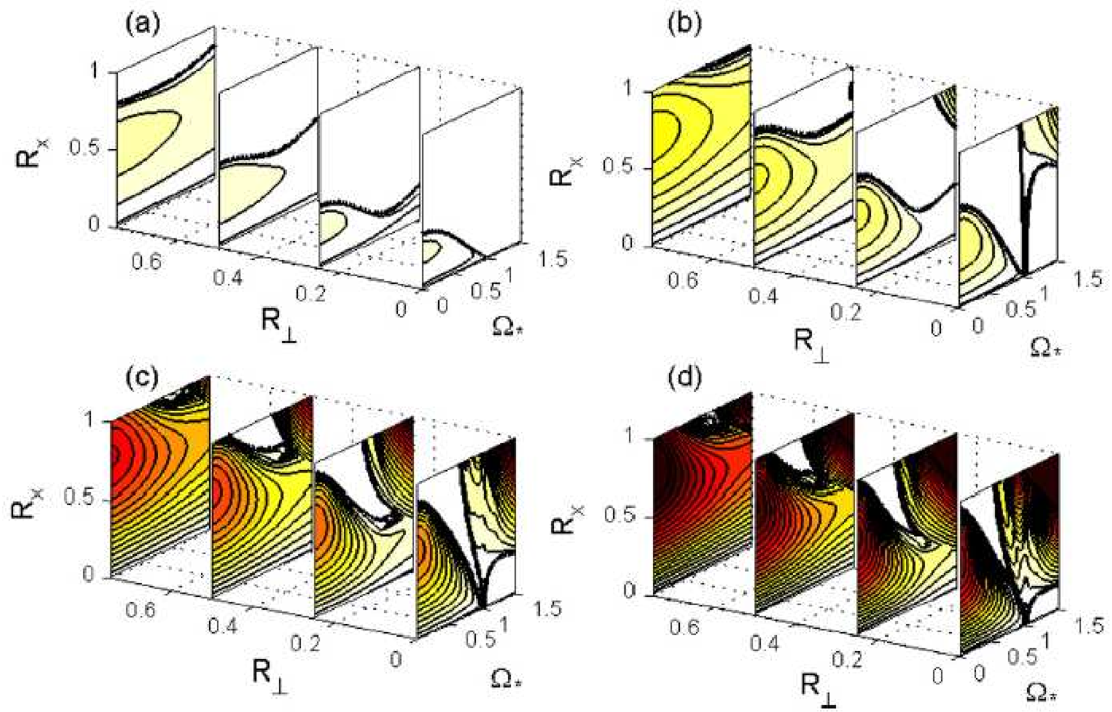
$$+ \phi_- e^{i[K_x x - \vec{k}\cdot\vec{x} - (\Omega - \Omega_0)t]} +$$

$$+ \phi_+ e^{i[K_x x + \vec{k}\cdot\vec{x} - (\Omega + \Omega_0)t]} + c.c.$$

Dispersion relation - Zonal flow

- The zonal flow dispersion relation of the 9 secondary instabilities depends on 6 dimensionless parameters.

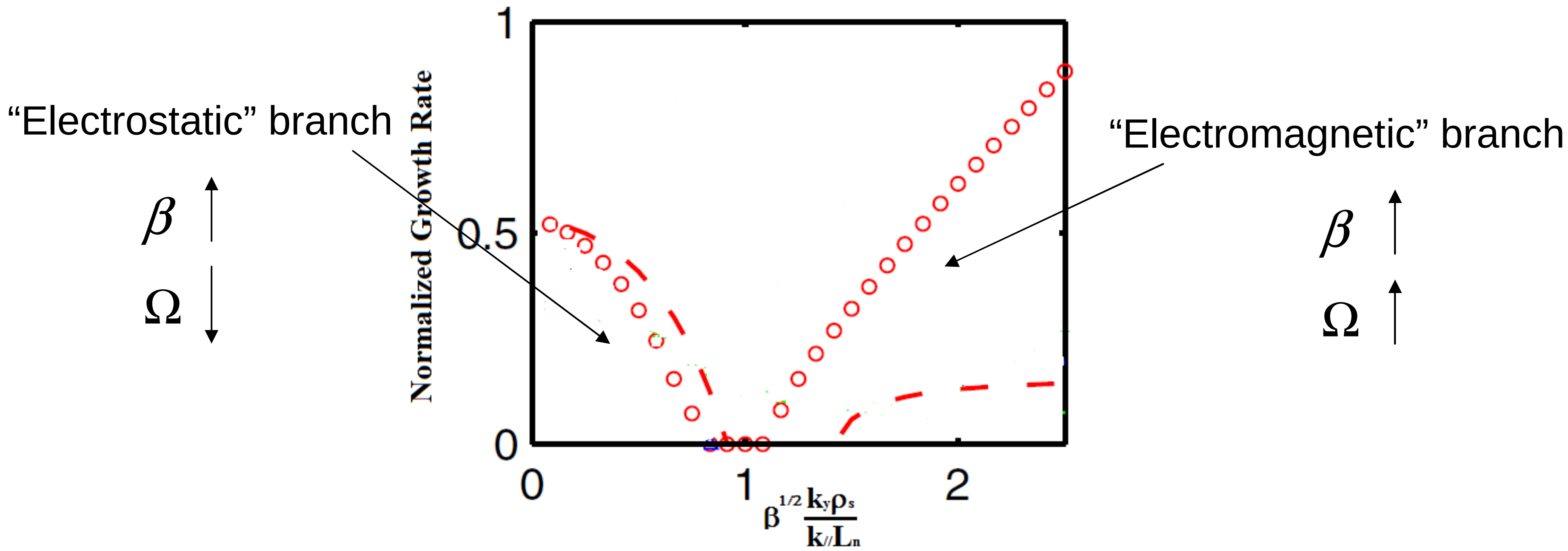
$$\Omega = \Omega(6\,\text{dimensionless parameters})$$



- Only one is particularly significant: $\sqrt{\beta}\,\frac{k_y \rho_s}{k_\parallel L_n}$

Feedback mechanism

- With respect to this parameter, the zonal flow growth rate has a peculiar behavior.
- Negative feedback mechanism in the "electrostatic" branch
- Positive feedback mechanism in the "electromagnetic" branch.



Self-sustained Zonal Flows

- As noted before, self-sustained Zonal Flows appear for

$$\sqrt{\beta}\frac{k_y \rho_s}{k_\parallel L_n} > 1$$

  for a wide range of the other 5 parameters.
- At resonant surfaces $k_{//}$=0 and this condition is always verified.
- Expanding $k_{//}$ around a resonant surface we find:

$$k_{//} = k_y \frac{x}{L_s}$$

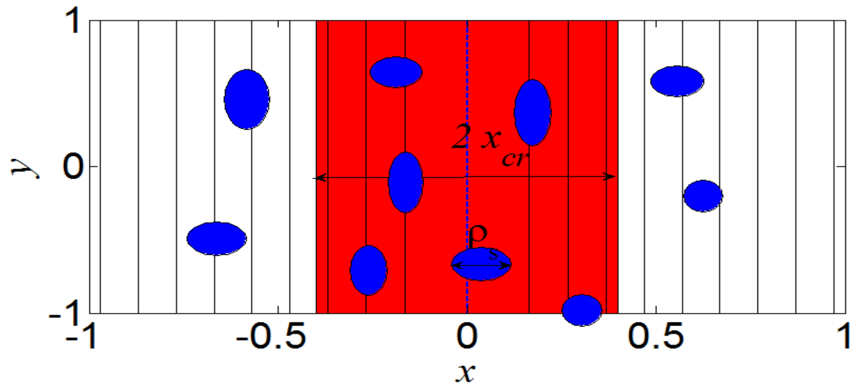
- We can identify an x_{cr} such that the zonal flows are self-sustained for $x<x_{cr}$

ITB criterion

- We can define a width around the surface where the condition is always met:

$$\sqrt{\beta}\frac{L_s}{L_n}\frac{\rho_s}{x} > 1 \quad \Rightarrow \quad x < x_{cr} \equiv \rho_s \sqrt{\beta}\frac{L_s}{L_n}$$

- To shear the eddies, x_{cr} must be larger than ρ_s:



$$x_{cr} \geq \rho_s \Rightarrow \sqrt{\beta}\frac{L_s}{L_n} \geq 1$$ or equivalently: $$\rho_{*M} \equiv \left(\frac{L_s}{d_i}\right)\frac{\rho_s}{L_n} \geq 1$$

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Conclusions

$$\frac{\rho_s}{L_n} \geq \frac{d_i}{L_s}$$

- Comparison between theory and experiment:

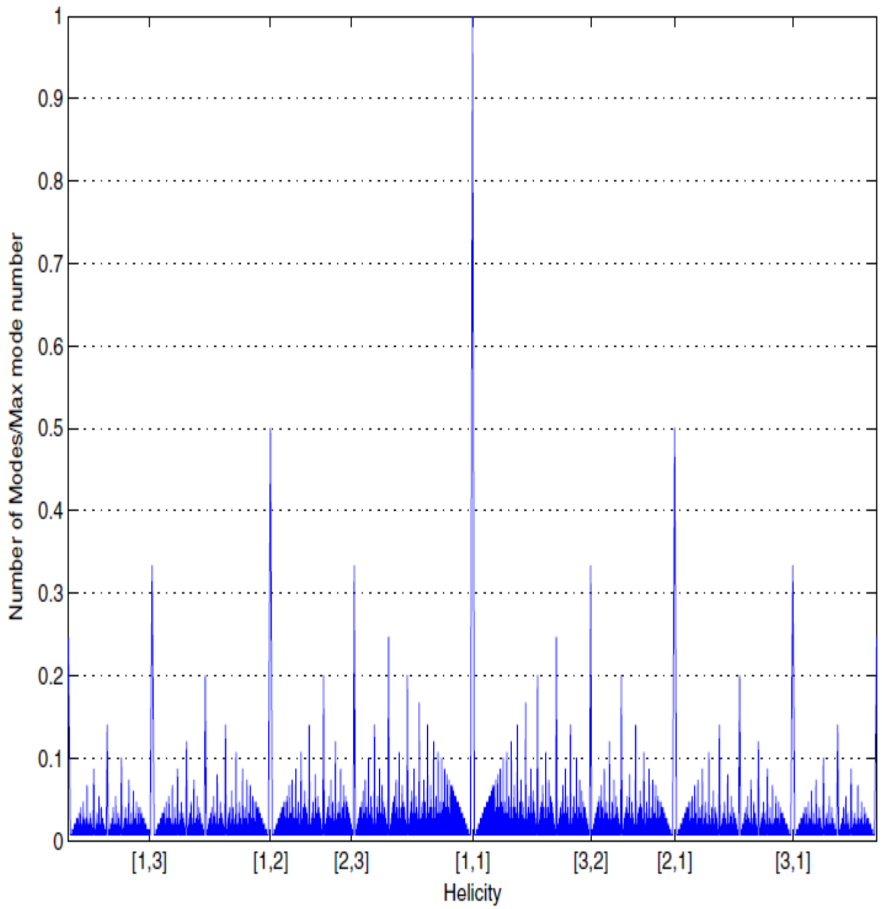
1. Low order rational surface needed to generate the zonal flows.
2. Low shear (i.e. large L_s) helps exceeding the threshold.
3. $\rho_* = \rho_s / L_n$ has to be greater than a threshold

ITB criterion II

- The criterion is a **necessary** condition but it is not **sufficient** (it does not say anything about the nonlinear evolution).
- Assumption: a barrier survives if a sufficient amount of energy goes from the turbulence to the zonal perturbation.
- Only the resonant modes transfer energy to the zonal perturbation.
- Low order helicities have more resonant modes.
- Conclusion:

an ITB forms and survives when a low order rational surface enters the palsma and:

$$\rho_{*M} \equiv \left(\frac{L_s}{d_i}\right)\frac{\rho_s}{L_n} \geq 1$$



Summary

- Using a minimal first principle model we have found a criterion for the ITB formation.
- Drift waves are used as a proxy for core turbulence modes.
- Self sustained Zonal Flows, which lead to the ITB, arise when $\sqrt{\beta}\frac{k_y \rho_s}{k_\parallel L_n}$ is grater than unity.
- The criterion only tells when the barrier should form. It does not say anything about its nonlinear evolution or the value of the transport coefficients.
- Details in: *F. Militello, M. Romanelli, J.W. Connor and R.J. Hastie, Nuclear Fusion 51, 033006 (2011)*.